CONVENIENT TOPOLOGY

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Abstract. A new viewpoint of Topology, summarized under the name Convenient Topology, is considered in such a way that the structural deficiencies of topological and uniform spaces are remidied. This does not mean that these spaces are superfluous. It means exactly that a better framework for handling problems of a topological nature is used. In this context semiuniform convergence spaces play an essential role. They include not only convergence structures such as topological structures and limit space structures, but also uniform convergence structures such as uniform structures and uniform limit space structures, and they are suitable for studying continuity, Cauchy continuity and uniform continuity as well as convergence structures in function spaces, namely simple convergence, continuous convergence and uniform convergence. Several results are presented which cannot be obtained by using topological or uniform spaces respectively.

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1. Introduction

Up to the middle of the thirty's mainly topological spaces have been studied in Topology. Some years later also uniform spaces (cf. [41] and [40]) have been investigated and thus concepts such as completeness, uniform continuity and uniform convergence were no longer excluded from consideration in a more general context than that one of metric spaces. But neither topological spaces nor uniform spaces are always well behaved with respect to the formation of function spaces, namely in 1946 Arens [2; Satz 3] proved that on the set $C(\mathbb{R}^{\mathbb{N}}, [0,1])$ of all continuous maps from the (usual) topological space $\mathbb{R}^{\mathbb{N}}$ of all sequences of real numbers into the closed unit interval $[0,1]$ there is no coarsest topology such that the evaluation map $ev : \mathbb{R}^{\mathbb{N}} \times C(\mathbb{R}^{\mathbb{N}}, [0,1]) \to [0,1]$, defined by $ev((x_n), f)) = f((x_n))$, is continuous. In other words: On the set $C(\mathbb{R}^{\mathbb{N}}, [0,1])$ there is no topology describing continuous convergence, originally defined by Hahn [17] in 1921. Thus, for infinite–dimensional Analysis continuous convergence cannot be described in the realm of topological spaces. The situation for uniform spaces is similar, namely it is not hard to prove that on the set $U(\mathbb{R}_u, \mathbb{R}_u)$ of all uniformly continuous maps from the usual uniform space $\mathbb{R}_u$ of real numbers into itself there is no uniformity such that the evaluation map $ev : \mathbb{R}_u \times U(\mathbb{R}_u, \mathbb{R}_u) \to \mathbb{R}_u$ is uniformly continuous (cf. e.g. [4]).

In the fifty's Kowalsky [25] and Fischer [14] (independently) introduced limit spaces (also called convergence spaces) as a generalization of topological spaces. In 1965 their usefulness for studying continuous convergence became apparent (cf. [8]).

In the language of category theory the above mentioned results mean that the category **Lim** of limit spaces (and continuous maps) is cartesian closed whereas the category **Top** of topological spaces (and continuous maps) [resp. **Unif** of uniform spaces (and uniformly continuous maps)] is not cartesian closed. According to Steenrod [39] desirable categories of spaces in Topology should be cartesian closed in order to be convenient. Since **Top** is nicely embedded into **Lim**, the question arises whether **Unif** can be nicely embedded into a cartesian closed (topological) category. This problem has been solved by Wyler [42] who proposed to weaken one of the axioms for uniform limit spaces introduced by Cook and Fischer [9] in 1967 (cf. also [26]). The resulting category is denoted by **ULim**.

Besides cartesian closedness other convenient properties of topological constructs have been studied later on, namely

1. Extensionality [= hereditariness] (cf. [22]);

and

2. Arbitrary products of quotients are quotients.

Roughly speaking topological constructs are categories of structured sets (and structure preserving maps) in which initial and final structures are available, e.g. products, subspaces, sums (=coproducts) and quotient spaces can be formed. A cartesian closed and extensional topological construct is also called a topological universe, in other words: A topological universe is a topological construct which is a quasitopos in the sense of M.J. Penon [27]. Extensionality means that final sinks are hereditary, i.e. in particular that quotients are hereditary. This is not true in **Top** in general (note: In 1963 Arhangels'kii [3] described the hereditary quotient maps in **Top** [i.e. those quotient maps $f : X \to Y$ in **Top** such that for each subspace $B \subset Y$, the restriction $f\big|_{f^{-1}[B]} : f^{-1}[B] \to B$ is also a quotient map] as the pseudo–open maps [i.e. those surjective continuous maps $f : X \to Y$ such that a point $y \in Y$ belongs to the interior of $f[U]$ for any neighborhood U of $f^{-1}(y)$]), but it is true in **Lim**. Hereditariness of quotients plays an essential role in the theory of connection and disconnection (cf. [35] and [28; 4.1]). In cartesian closed topological constructs quotients are always finitely productive but they need neither be countably productive nor hereditary as the topological construct **Chy** of Cauchy spaces (and Cauchy continuous maps) shows (cf. [6]). A topological universe $\mathcal{C}$ is called strong provided that in $\mathcal{C}$ products of quotients are quotients. **Lim** is a strong topological universe whereas **ULim** is not even a topological universe (cf. [4]).

Many attempts have been made in the past to embed topological and uniform spaces into a common topological superconstruct (e.g. quasiuniform spaces by Nachbin (cf. [15]), syntopogeneous spaces by Császár [10], generalized topological spaces (= supertopological spaces) by Doitchinov [11], merotopic spaces (= seminearness spaces) by Katětov [24] and nearness spaces by Herrlich [19]), but none of them led to a cartesian closed topological construct.

The aim of Convenient Topology consists in the study of strong topological universes in which convergence structures (e.g. **Lim**–structures) and uniform convergence structures

(e.g. **ULim**–structures) are available. Furthermore, such a strong topological universe should be easily described by means of suitable axioms and should not be to big. Omitting two of the defining axioms of uniform limit spaces one obtains semiuniform convergence spaces and the topological construct **SUConv** of semiuniform convergence spaces (and uniformly continuous maps) fulfills the above mentioned criteria. Thus, in the realm of Convenient Topology we are mainly concerned with semiuniform convergence spaces or more exactly with the study of **SUConv**–invariants, i.e. properties of semiuniform convergence spaces which are preserved by isomorphisms in **SUConv** (this includes the study of full and isomorphism–closed subconstructs of **SUConv**). Therefore, the study of (symmetric) topological spaces and uniform spaces belongs to Convenient Topology, but it includes also the study of many other important topological constructs such as **ULim**, $\mathbf{KConv_s}$ (= construct of symmetric Kent convergence spaces), **Fil** (= construct of filter spaces in the sense of Katětov [24]), **Chy** and **Prox** (= construct of proximity spaces in the sense of Efremovič [12]). (cf. the diagram under 2.10).
The terminology of this article corresponds to [1] and [29].

Conventions. 1) A filter on a set X is not allowed to contain the empty set $\emptyset$.
2) Subcategories are always assumed to be full and isomorphism–closed.

2. Topological constructs

2.1. By a construct we mean a category $\mathcal{C}$ whose objects are structured sets, i.e. pairs (X,ξ) where X is a set and ξ a $\mathcal{C}$–structure on X, whose morphisms $f:(X,\xi)\to(Y,\eta)$ are suitable maps between X and Y and whose composition law is the usual composition of maps.

2.2 Definition. 1) A construct $\mathcal{C}$ is called *topological* iff it satisfies the following conditions:

(1) *Existence of initial structures.*
For any set X, any family $((X_i,\xi_i))_{i\in I}$ of $\mathcal{C}$–objects indexed by a class I and any family $(f_i : X\to X_i)_{i\in I}$ of maps indexed by I, there exists a unique $\mathcal{C}$–structure ξ on X which is *initial* with respect to $(X, f_i, (X_i,\xi_i), I)$, i.e. such that for any $\mathcal{C}$–object (Y,η) a map $g:(Y,\eta)\to(X,\xi)$ is a $\mathcal{C}$–morphism iff for every $i\in I$ the composite map $f_i\circ g:(Y,\eta)\to(X_i,\xi_i)$ is a $\mathcal{C}$–morphism.

(2) For any set X, the class of all $\mathcal{C}$–structures on X is a set.

(3) For any set X with cardinality at most one, there exists exactly one $\mathcal{C}$–structure on X.

2.3 Proposition. *For a construct $\mathcal{C}$ the following are equivalent:*

(1) $\mathcal{C}$ satisfies (1) in 2.2.

(2) *For any set X, any family $((X_i,\xi_i))_{i\in I}$ of $\mathcal{C}$–objects indexed by some class I and any family $(f_i, X_i \to X)_{i\in I}$ of maps indexed by I, there exists a unique $\mathcal{C}$–struture ξ on X which is final with respect to $((X_i,\xi_i), f_i, X, I)$, i.e. for any $\mathcal{C}$–object (Y,η), a map $g : (X,\xi) \to (Y,\eta)$ is a $\mathcal{C}$–morphism iff for every $i \in I$ the composite map $g \circ f_i : (X_i,\xi_i) \to (Y,\eta)$ is a $\mathcal{C}$–morphism.*

Proof. cf. [29;1.2.1.1.]

2.4 Remarks. 1) If $\mathcal{C}$ is a topological construct and X is a set, then the set $\mathcal{C}_X$ of all $\mathcal{C}$–structures on X, ordered by $\xi \le \eta \Leftrightarrow 1_X : (X,\xi) \to (X,\eta)$ is a $\mathcal{C}$–morphism, is a complete lattice. ξ is called *finer* then η (and η *coarser* then ξ) provided that $\xi \le \eta$.

2) Let $\mathcal{C}$ be a topological construct. Then the initial (resp. final) $\mathcal{C}$–structure on a set X with respect to $(X, f_i, (X_i,\xi_i), I)$ (resp. $((X_i,\xi_i), f_i, X, I)$) is the coarsest (resp. finest) $\mathcal{C}$–structure on X such that each $f_i : X \to X_i$ (resp. $f_i : X_i \to X$) is a $\mathcal{C}$–morphism.

3) Since in a topological construct $\mathcal{C}$, initial and final structures are available, products, subspaces, sums (= coproducts) and quotient spaces can be formed in $\mathcal{C}$. It is easily checked that in a topological construct *sums of quotients are quotients (of sums), but products of quotients need not be quotients (of products)* (cf. e.g. [13;2.4.20]).

2.5 Examples of topological constructs.

1) The construct **Top** of topological spaces (and continuous maps).
2) The construct **Unif** of uniform spaces (and uniformly continuous maps).
3) The construct **Prox** of proximity spaces (and δ–maps).
4) The construct **GConv**, **KConv**, **Lim**, **PsTop** and **PrTop** of generalized convergence spaces, Kent convergence spaces, limit spaces, pseudotopological spaces and pretopological spaces (and continuous maps) respectively.
(Let X be a set, $F(X)$ the set of all filters on X and $q \subset F(X) \times X$. Consider the following conditions:

C_1) $(\dot{x}, x) \in q$ for each $x \in X$, where $\dot{x} = \{A \subset X : x \in A\}$.

C_2) $(\mathcal{G}, x) \in q$ whenever $(\mathcal{F}, x) \in q$ and $\mathcal{F} \subset \mathcal{G}$.

C_3) $(\mathcal{F} \cap \dot{x}) \in q$ whenever $(\mathcal{F}, x) \in q$.

C_4) $(\mathcal{F} \cap \mathcal{G}, x) \in q$ whenever $(\mathcal{F}, x) \in q$ and $(\mathcal{G}, x) \in q$.

C_5) $(\mathcal{F}, x) \in q$ whenever $(\mathcal{G}, x) \in q$ for each ultrafilter $\mathcal{G} \supset \mathcal{F}$.

C_6) $(\mathcal{U}_q(x), x) \in q$ where $\mathcal{U}_q(x) = \bigcap\{\mathcal{F} : (\mathcal{F}, x) \in q\}$.

Then (X,q) is called a *generalized convergence space* if C_1) and C_2) are satisfied, a *Kent convergence space* if C_1), C_2) and C_3) are satisfied, a *limit space* if C_1), C_2) and C_4) are satisfied, a *pseudotopological space* or *Choquet space* if C_1), C_2) and C_5) are satisfied, and a *pretopological space* if C_1), C_2) and C_6) are satisfied.

Instead of $(\mathcal{F}, x) \in q$ one usually writes $\mathcal{F} \to x$ (read: $\mathcal{F}$ converges to x). In each case the morphisms are all continuous maps, i.e. those maps carrying filters converging to x to filters converging to $f(x)$.)
5) The constructs $\mathbf{GConv_s}$, $\mathbf{KConv_s}$, $\mathbf{Lim_s}$, $\mathbf{PsTop_s}$, $\mathbf{PrTop_s}$ and $\mathbf{Top_s}$ of symmetric generalized convergence spaces, symmetric Kent convergence spaces, symmetric limit spaces, symmetric pseudotopological spaces, symmetric pretopological spaces and symmetric topological spaces (and continuous maps) respectively (A generalized convergence space (X, q) is called *symmetric* provided that the following is satisfied:

(S) $(\mathcal{F}, x) \in q$ and $y \in \cap\mathcal{F}$ imply $(\mathcal{F}, y) \in q$.

In particular, a topological space X is symmetric if and only if it is an R_0–space , i.e. $x \in \overline{\{y\}}$ implies $y \in \overline{\{x\}}$ for each $(x, y) \in X \times X$).
6) The construct **Fil** (resp. **Chy**) of filter spaces (resp. Cauchy spaces) and Cauchy continuous maps.
(Let X be a set and $\gamma \subset F(X)$. Consider the following conditions

(1) $\dot{x} \in \gamma$ for each $x \in X$.

(2) $\mathcal{G} \in \gamma$ whenever $\mathcal{F} \in \gamma$ and $\mathcal{F} \subset \mathcal{G}$.

(3) If $\mathcal{F}$ and $\mathcal{G}$ belong to γ such that every member of $\mathcal{F}$ meets every member of $\mathcal{G}$, then $\mathcal{F} \cap \mathcal{G}$ belongs to γ.

Then (X, γ) is called a *filter* space (resp. *Cauchy space*) if (1) and (2) (resp. (1), (2) and (3)) are satisfied and the elements of γ are called *Cauchy filters*. A map $f : (X, \gamma) \to (X', \gamma')$ between filter spaces is called *Cauchy continuous* provided that $f(\mathcal{F}) \in \gamma'$ for each $\mathcal{F} \in \gamma$.)
7) The constructs **SUConv**, **SULim** and **ULim** of semiuniform convergence spaces, semiuniform limit spaces and uniform limit spaces (and uniformly continuous maps) respectively.
(Let X be a set and $\mathcal{J}_X \subset F(X \times X)$. Consider the following conditions:

UC_1) $\dot{x} \times \dot{x} \in \mathcal{J}_X$ for each $x \in X$, where $\dot{x} \times \dot{x} = \{A \subset X \times X : (x, x) \in A\}$.

UC_2) $\mathcal{F} \in \mathcal{J}_X$ whenever $\mathcal{G} \in \mathcal{J}_X$ and $\mathcal{G} \subset \mathcal{F}$.

UC_3) $\mathcal{F} \in \mathcal{J}_X$ implies $\mathcal{F}^{-1} = \{F^{-1} : F \in \mathcal{F}\} \in \mathcal{J}_X$, where $F^{-1} = \{(x, y) \in X \times X : (y, x) \in F\}$.

UC_4) $\mathcal{F} \in \mathcal{J}_X$ and $\mathcal{G} \in \mathcal{J}_X$ imply $\mathcal{F} \cap \mathcal{G} \in \mathcal{J}_X$.

UC_5) $\mathcal{F} \in \mathcal{J}_X$ and $\mathcal{G} \in \mathcal{J}_X$ imply $\mathcal{F} \circ \mathcal{G} \in \mathcal{J}_X$ (whenever $\mathcal{F} \circ \mathcal{G}$ exists, i.e. $F \circ G = \{(x, y) : \exists z \in X$ with $(x, z) \in G$ and $(z, y) \in F\} \neq \phi$ for every $F \in \mathcal{F}, G \in \mathcal{G}$), where $\mathcal{F} \circ \mathcal{G}$ is the filter generated by the filter base $\{F \circ G : F \in \mathcal{F}$ and $G \in \mathcal{G}\}$.

Then $(X, \mathcal{J}_X)$ is called a *semiuniform convergence space* provided that $UC_1)$, $UC_2)$ and $UC_3)$ are fulfilled. A semiuniform convergence space $(X, \mathcal{J}_X)$ is called a *semiuniform limit space* provided that $UC_4)$ is satisfied. A semiuniform limit space satisfying $UC_5)$ is called a *uniform limit space.*
A map $f : (X, \mathcal{J}_X) \to (X, \mathcal{J}_Y)$ between semiuniform convergence spaces is called *uniformly continuous* provided that $(f \times f)(\mathcal{J}_X) \subset \mathcal{J}_Y$, i.e. $(f \times f)(\mathcal{F}) \in \mathcal{J}_Y$ for each $\mathcal{F} \in \mathcal{J}_X$.
Let X be a set and $(X_i, \mathcal{J}_{X_i}))_{i \in I}$ a family of semiuniform convergence spaces, semiuniform limit spaces or uniform limit spaces respectively. Then $\mathcal{J}_X = \{\mathcal{F} \in F(X \times X) :$
$(f_i \times f_i)(\mathcal{F}) \in \mathcal{J}_{X_i}$ for each $i \in I\}$ is the initial **SUConv**–, **SULim**– or **ULim**–structure respectively w.r.t. the given data.)

2.6. In the following, subconstructs $\mathcal{A}$ of a topological construct $\mathcal{C}$ are always assumed to be

1. *full*, i.e. for each pair (A, B) of $\mathcal{A}$–objects the set of all morphisms between A and B is in $\mathcal{A}$ and in $\mathcal{C}$ the same.

and

2. *isomorphism-closed*, i.e. each $\mathcal{C}$–object being isomorphic to an $\mathcal{A}$–object is an $\mathcal{A}$-object.

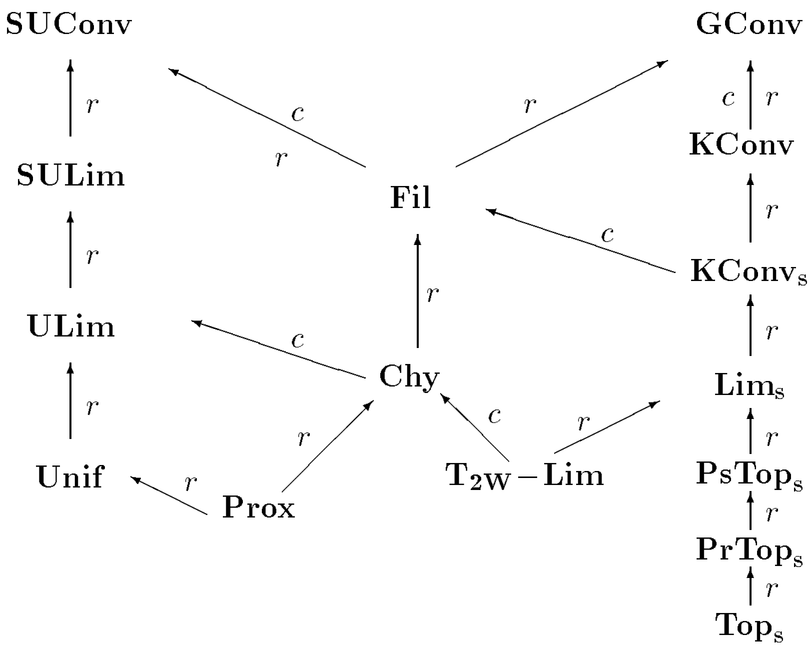
In order to describe the relationship between most of the above mentioned topological constructs the following definition is useful.

2.7 Definition. A subconstruct $\mathcal{A}$ of a topological construct $\mathcal{C}$ is called *bireflective* (resp. *bicoreflective*) provided that for each $\mathcal{C}$–object (X, ξ) there is a [unique] $\mathcal{A}$–structure $\xi_{\mathcal{A}}$ on X which is coarser than ξ (resp. finer than ξ) such that for each $\mathcal{A}$–object (Y, η) and each $\mathcal{C}$–morphism $f : (X, \xi) \to (Y, \eta)$ (resp. $f : (Y, \eta) \to (X, \xi)$), $f : (X, \xi_{\mathcal{A}}) \to (Y, \eta)$ (resp. $f : (Y, \eta) \to (X, \xi_{\mathcal{A}})$) is a $\mathcal{C}$–morphism. $\xi_{\mathcal{A}}$ is called the *bireflective* (resp. *bicoreflective*) *$\mathcal{A}$–modification* of the $\mathcal{C}$–structure ξ.

2.8 Remark. The bireflective (resp. bicoreflective) $\mathcal{A}$–modificatin $\xi_{\mathcal{A}}$ of the $\mathcal{C}$–structure ξ [cf. 2.7.] is the finest one (resp. coarsest one) of all $\mathcal{C}$–structure ξ' on X which are coarser (resp. finer) than ξ and for which $(X, \xi') \in |\mathcal{A}|$ (cf. [29;2.2.11]).

2.9 Proposition (cf. [29;2.2.12 and 2.2.13.]). *Let $\mathcal{A}$ be a subconstruct of a topological construct. Then $\mathcal{A}$ is topological provided that $\mathcal{A}$ is bireflective (resp. bicoreflective) in $\mathcal{C}$. Particularly, if $\mathcal{A}$ is bireflective (resp. bicoreflective) in $\mathcal{C}$, then the initial structures (resp. final structures) in $\mathcal{A}$ are formed as in $\mathcal{C}$, whereas the final structures (resp. initial structures) arise from the final structures (resp. initial structures) in $\mathcal{C}$ by bireflective (resp. bicoreflective) $\mathcal{A}$–modification.*

2.10. In the following diagram r (resp. c) stands for embedding as a bireflective (resp. bicoreflective) subconstruct (cf. [31;9]).



(**T$_{2W}$–Lim** denotes the construct of weakly Hausdorff limit spaces and continuous maps, where a limit space (X, q) is called *weakly Hausdorff* [or T_{2W}] provided that the existence of a filter $\mathcal{F}$ on X converging to $x, y \in X$ implies $\{\mathcal{G} \in F(X) : (\mathcal{G}, x) \in q\} = \{\mathcal{H} \in F(X) : (\mathcal{H}, y) \in q\}$).

2.11 Remarks. 1) If $(X, \mathcal{J}_X)$ is a semiuniform convergence space, then the bicoreflective **Fil**–modification $\gamma_{\mathcal{J}_X}$ of $\mathcal{J}_X$ is described by

$$\mathcal{F} \in \gamma_{\mathcal{J}_X} \Longleftrightarrow \mathcal{F} \times \mathcal{F} \in \mathcal{J}_X \; .$$

$(X, \gamma_{\mathcal{J}_X})$ is called *the underlying filter space* of $(X, \mathcal{J}_X)$.

2)a) If (X, γ) is a filter space, then the bicoreflective **KConv$_s$**–modification of γ is described by

$$(\mathcal{F}, x) \in q_\gamma \Longleftrightarrow \mathcal{F} \cap \dot{x} \in \gamma \; .$$

b) If $(X, \mathcal{J}_X)$ is a semiuniform convergence space, then $(X, q_{\gamma_{\mathcal{J}_X}})$ is called the *underlying (symmetric) Kent convergence space (shortly: the underlying convergence space)* of $(X, \mathcal{J}_X)$.

3. Natural function spaces

3.1 Definitions. 1) A topological construct $\mathcal{C}$ is called *cartesian closed* provided that for any pair (A, B) of $\mathcal{C}$–ojects the set $[A, B]_{\mathcal{C}}$ of all $\mathcal{C}$–morphisms from A to B can be endowed with the structure of a $\mathcal{C}$–object denoted by B^A and called *power object* or *natural function space* such that the following are satisfied:

(1) The evaluation map $e_{A,B} : A \times B^A \to B$ defined by $e_{A,B}(a, g) = g(a)$ for each $(a, g) \in A \times B^A$ is a $\mathcal{C}$–morphism.

(2) For each $\mathcal{C}$–object C and each $\mathcal{C}$–morphism $f : A \times C \to B$ the map $\overline{f} : C \to B^A$ defined by $\overline{f}(c)(a) = f(a,c)$ is a $\mathcal{C}$–morphism.

2) A class–indexed family $(f_i : (X_i, \xi_i) \to (X, \xi))_{i \in I}$ of morphisms in a topological construct $\mathcal{C}$ is called a *final epi–sink* provided that $X = \bigcup_{i \in I} f_i[X_i]$ and ξ is the final $\mathcal{C}$–structure with respect to $((X_i, \xi_i), f_i, X, I)$.

3.2 Theorem. (cf. [18] or [29;4.1.4.]). *Let $\mathcal{C}$ be a topological construct. Then the following are equivalent:*

(1) $\mathcal{C}$ is cartesian closed.

(2) For any $A \in |\mathcal{C}|$ and any set–indexed family $(B_i)_{i \in I}$ of $\mathcal{C}$–objects, the following are satisfied:

(a) $A \times \coprod_{i \in I} B_i \cong \coprod_{i \in I} A \times B_i$.

(b) If f is a quotient map, then so ist $1_A \times f$.

(3)(a) For any $A \in |\mathcal{C}|$ and any set–indexed family $(B_i)_{i \in I}$ of $\mathcal{C}$–objects the following is satisfied:

$$A \times \coprod_{i \in I} B_i \cong \coprod_{i \in I} (A \times B_i)$$

(b) In $\mathcal{C}$ the product $f \times g$ of any two quotient maps f and g is a quotient map.

(4) For each $\mathcal{C}$–object A holds: For any final epi–sink $(f_i : B_i \to B)_{i \in I}$ in $\mathcal{C}$, $(1_A \times f_i : A \times B_i \to A \times B)_{i \in I}$ is a final epi–sink.

3.3 Corollary (cf. [1;27.8] or [29;4.1.5.]). *Let $\mathcal{C}$ be a cartesian closed topological construct. Then the following are satisfied:*

(1) First exponential law: $A^{B \times C} \cong (A^B)^C$

(2) Second exponential law: $(\prod_{i \in I} A_i)^B \cong \prod_{i \in I} A_i^B$

(3) Third exponential law: $A^{\coprod_{i \in I} B_i} \cong \prod_{i \in I} A^{B_i}$

(4) Distributive law: $A \times \coprod_{i \in I} B_i \cong \coprod_{i \in I} A \times B_i$.

3.4 Corollary (cf. [1;27.9.(2)]). *Let $\mathcal{C}$ be a cartesian closed topological construct and $\mathcal{A}$ a bicoreflective subconstruct which is closed under formation of finite products in $\mathcal{C}$. Then $\mathcal{A}$ is cartesian closed, and the power–objects in $\mathcal{A}$ arise from the corresponding power–objects in $\mathcal{C}$ by bicoreflective $\mathcal{A}$–modification of their underlying $\mathcal{C}$–structures.*

3.5 Remark. It is easily checked that a bireflective subconstruct $\mathcal{A}$ of a topological construct $\mathcal{C}$, which is closed under formation of power–objects in $\mathcal{C}$, is cartesian closed, and that the power–objects in $\mathcal{A}$ are formed as in $\mathcal{A}$.

3.6 Proposition. *Let $\mathcal{C}$ be a cartesian closed topological construct. If A and B are $\mathcal{C}$–*

objects and $([A,B]_{\mathcal{C}},\xi)$ *is the natural function space, then* ξ *is the coarsest* $\mathcal{C}$*–structure for which the evaluation map* $e_{A,B} : A \times ([A,B]_{\mathcal{C}},\xi) \to B$ *is a* $\mathcal{C}$*–morphism.*

Proof. Apply 3.1.1) (2) to $f = e_{A,B}$.

3.7 Examples. 1) **Top** is not cartesian closed, because in **Top** quotient maps are not finitely productive (cf. [13;2.4.20]).

2) **Unif** is not cartesian closed (cf. [4;1.5.5]).

3)a) **SUConv** is cartesian closed: Let $\mathbf{X} = (X, \mathcal{J}_X)$ and $\mathbf{Y} = (Y, \mathcal{J}_Y)$ be semiuniform convergence spaces. Then the power object $\mathbf{Y}^{\mathbf{X}}$ is the set $[\mathbf{X},\mathbf{Y}]_{\mathbf{SUConv}}$ of all uniformly continuous maps from $\mathbf{X}$ into $\mathbf{Y}$ endowed with the **SUConv**–structure $\mathcal{J}_{X,Y} = \{\phi \in F([\mathbf{X},\mathbf{Y}]_{\mathbf{SUConv}} \times [\mathbf{X},\mathbf{Y}]_{\mathbf{SUConv}}) : \phi(\mathcal{F}) \in \mathcal{J}_Y \text{ for each } \mathcal{F} \in \mathcal{J}_X\}$ where $\phi(\mathcal{F})$ is the filter generated by the filter base $\{A(F) : A \in \phi, F \in \mathcal{F}\}$ with $A(F) = \{(f(a), g(b)) : (f,g) \in A, (a,b) \in F\}$. $\mathcal{J}_{X,Y}$ is called *the uniformly continuous* **SUConv**–*structure.*

b) Since **SULim** (resp. **ULim**) is bireflective in **SUConv** and closed under formation of power–objects in **SUConv**, it is cartesian closed (cf. 3.5.)

4) **Fil** and **Chy** are cartesian closed: Let $\mathbf{X} = (X,\gamma)$ and $\mathbf{X}' = (X',\gamma')$ be filter spaces (resp. Cauchy spaces). Then the power–object $\mathbf{X}'^{\mathbf{X}}$ is the set $[\mathbf{X},\mathbf{X}']$ of all Cauchy continuous maps from $\mathbf{X}$ into $\mathbf{X}'$ endowed with the **Fil**–structure (resp. **Chy**–structure) $\widehat{\gamma} = \{\Theta \in F([\mathbf{X},\mathbf{X}']) : \Theta(\mathcal{F}) \in \gamma' \text{ for } \mathcal{F} \in \gamma\}$, where $\Theta(\mathcal{F})$ is the filter generated by the filter base $\{A(F) : A \in \Theta, F \in \mathcal{F}\}$ with $A(F) = \{f(x) : f \in A, x \in F\}$. $\widehat{\gamma}$ is called the *Cauchy continuous* **Fil**–*structure* (resp. **Chy**–*structure*).

5) **Lim** (resp. $\mathbf{Lim_s}$) is cartesian closed: If $\mathbf{X} = (X,q)$ and $\mathbf{X}' = (X',q')$ are limit spaces (resp. symmetric limit spaces), then the power–object $\mathbf{X}'^{\mathbf{X}}$ is the set $[\mathbf{X},\mathbf{X}']$ of all continuous maps from $\mathbf{X}$ into $\mathbf{X}'$ endowed with the **Lim**–structure (resp. $\mathbf{Lim_s}$–structure) $\hat{q}$ defined by

$$(\psi, f) \in \hat{q} \Leftrightarrow (\psi(\mathcal{F}), f(x)) \in q' \text{ for each } (\mathcal{F}, x) \in q ,$$

where $\psi(\mathcal{F}) = e_{\mathbf{X},\mathbf{X}'}(\mathcal{F} \times \psi)$, i.e. $\psi(\mathcal{F})$ is the filter generated by the filter base $\{A(F) : A \in \psi, F \in \mathcal{F}\}$, where $A(F) = \{f(z) : f \in A, z \in F\}$. $\hat{q}$ is called the **Lim**–*structure* (resp. $\mathbf{Lim_s}$–*structure*) *of continuous convergence.*

6) **KConv** (resp. $\mathbf{KConv_s}$) is cartesian closed, but the natural function space structure is not the structure of continuous convergence (cf. [36;p. 145])

3.8 Theorem (cf. [31;7.2.]). A) *Let* (X,γ) *(resp.* (X',γ')*) be a filter space and* $(X,\mathcal{J}_\gamma)$ *(resp.* $(X',\mathcal{J}_{\gamma'})$*) the corresponding semiuniform convergence space, i.e.* $\mathcal{J}_\gamma = \{\mathcal{F} \in F(X \times X) : \exists \mathcal{G} \in \gamma \text{ with } \mathcal{G} \times \mathcal{G} \subset \mathcal{F}\}$. *Then the Cauchy continuous* **Fil**–*structure* $\hat{\gamma}$ *on* $[(X,\gamma),(X',\gamma')]_{\mathbf{Fil}}$ *is the bicoreflective* **Fil**–*modification of the uniformly continuous* **SUConv**–*structure* $\mathcal{J}_{X,X'}$ *on* $[(X,\mathcal{J}_\gamma),(X',\mathcal{J}_{\gamma'})]_{\mathbf{SUConv}}$.

B) *Let* (X,q) *(resp.* (X',q')*) be a symmetric limit space and* (X,γ_q) *(resp.* $(X',\gamma_{q'})$*) the corresponding filter space, i.e.* γ_q *[resp.* $\gamma_{q'}$*] consists of all convergent filters in* (X,q) *[resp.* (X',q')*]. Then the* $\mathbf{Lim_s}$–*strucure* $\hat{q}$ *of continuous convergence is* *the bicoreflective* $\mathbf{KConv_S}$*–modification of the Cauchy continuous* $\mathbf{Fil}$*–structure* $\hat{\gamma}$ *on*

$$[(X,\gamma_q),(X',\gamma_{q'})]_{\mathbf{Fil}} = [(X,q),(X',q')]_{\mathbf{KConv_S}} = [(X,q),(X',q')]_{\mathbf{Lim_S}} \ .$$

3.9 Remarks. 1) It follows from 3.8 that *the* $\mathbf{Lim_S}$*–structure of continuous convergence can be derived from the natural function space structure in* $\mathbf{SUConv}$*, namely from the uniformly continuous* $\mathbf{SUConv}$*–structure.*

2) The structures of *simple convergence* and *uniform convergence* can also be derived from the natural function space structure in $\mathbf{SUConv}$ (cf. [31;7.5.]).

4. Extensionality

4.1 Definitions. 1) In a topological construct $\mathcal{C}$, a *partial morphism* from A to B is a $\mathcal{C}$–morphism $f : C \to B$ whose domain is a subspace of A.

2) A topological construct $\mathcal{C}$ is called *extensional* (or *hereditary*) provided that every $\mathcal{C}$–object B has a *one–point extension* $B^* \in |\mathcal{C}|$, i.e. every $B \in |\mathcal{C}|$ can be embedded via the addition of a single point ∞_B into a $\mathcal{C}$–object B^* such that, for every partial morphism $f : C \to B$ from A to B, the map $f^* : A \to B^*$ defined by

$$f^*(a) = \begin{cases} f(a) & , \text{ if } \ a \in C \\ \infty_B & , \text{ if } \ a \notin C \end{cases} .$$

is a $\mathcal{C}$–morphism.

3) A final sink $(f_i : A_i \to A)_{i\in I}$ in a topological construct $\mathcal{C}$ is called *hereditary* provided that the following is satisfied: If B is a subspace of A, B_i a subspace of A_i with underlying set $f_i^{-1}[B]$ and $g_i : B_i \to B$ is the corresponding restriction of f_i, then $(g_i : B_i \to B)_{i\in I}$ is a final sink in $\mathcal{C}$ too.

4.2. Theorem (cf. [21]). *For a topological construct* $\mathcal{C}$ *the following are equivalent:*

(1) $\mathcal{C}$ *is extensional.*
(2) In $\mathcal{C}$ *final sinks are hereditary.*
(3) In $\mathcal{C}$ *final epi–sinks are hereditary.*
(4) Quotients and coproducts in $\mathcal{C}$ *(considered as final epi–sinks) are hereditary.*

4.3 Proposition (cf. [37;2.6.]). *Let* $\mathcal{C}$ *be an extensional topological construct and let* $(Y,\eta) \in |\mathcal{C}|$*. If* (Y^*,η^*) *denotes the one–point extension of* (Y,η)*, then* η^* *is the coarsest* $\mathcal{C}-$ *structure on* $Y^* = Y \cup \{\infty_Y\}$ *such that* (Y,η) *is a subspace of* (Y^*,η^*)*.*

4.4 Proposition (cf. [37;4.1.]). *Let* $\mathcal{C}$ *be an extensional topological construct and* $\mathcal{A}$ *a bicoreflective subconstruct which is closed under formation of subspaces in* $\mathcal{C}$*. Then* $\mathcal{A}$ *is extensional, and the one–point extensions in* $\mathcal{A}$ *arise from the one-point extensions in* $\mathcal{C}$ *by bicoreflective* $\mathcal{A}$*–modification of their underlying* $\mathcal{C}$*–structures.*

4.5 Remark. Obviously, if $\mathcal{A}$ is a bireflective subconstruct of an extensional topological construct $\mathcal{C}$, which is closed under formation of one–point extensions in $\mathcal{C}$, then $\mathcal{A}$ is an extensional topological construct, and the one–point extensions in $\mathcal{A}$ are formed as in $\mathcal{C}$.

4.6 Examples. 1) **Top** is not extensional (cf. e.g. [20;Thm.2])

2) **Unif** and **ULim** are not extensional (cf. e.g. [4;1.5.9.]).

3) **SUConv** is extensional: Let $(X, \mathcal{J}_X)$ be a semiuniform convergence space. Put $X^* = X \cup \{\infty_X\}$ with $\infty_X \notin X$. For each $M \subset X^* \times X^*$, let $M^* = M \cup (X^* \times \{\infty_X\}) \cup (\{\infty_X\} \times X^*)$. For each $\mathcal{F} \in \mathcal{J}_X$, consider the filter $\mathcal{F}^* = \{F^* : F \in \mathcal{F}\}$ on $X^* \times X^*$. Then $(X^*, \mathcal{J}_X^*)$ is the desired one–point extension of $(X, \mathcal{J}_X)$ provided that $\mathcal{J}_X^* = \{\mathcal{H} \in F(X^* \times X^*)$: there is some $\mathcal{F} \in \mathcal{J}_X$ with $\mathcal{F}^* \subset \mathcal{H}$ or $\{(\infty_X, \infty_X)\}^* \in \mathcal{H}\} \cup \{\dot{\infty}_X \times \dot{\infty}_X\}$.

4) **Fil** is extensional: Let (X, γ) be a filter space and $(X, \mathcal{J}_\gamma)$ its corresponding semiuniform convergence space (cf. 3.8.). If $(X^*, \mathcal{J}_\gamma^*)$ is the one–point extension of $(X, \mathcal{J}_\gamma)$ in **SUConv**, then (X^*, γ^*) is the one–point extension of (X, γ) in **Fil** provided that $\gamma^* = \gamma_{\mathcal{J}_\gamma^*}$)(cf. 2.7.1)).

5)a) **KConv** is extensional: Let (X, q) be a Kent convergence space. Put $X^* = X \cup \{\infty_X\}$ with $\infty_X \notin X$. Let $i : X \to X^*$ be the inclusion map. A **KConv**–structure q^* on X^* is defined by

$$(\mathcal{F}, x) \in q^* \Longleftrightarrow x = \infty_X \ \text{ or } \ \mathcal{F} = \dot{\infty}_X \ \text{ or } \ (i^{-1}(\mathcal{F}), x) \in q \ ,$$

i.e. all filters on X^* converge to ∞_X, and $\dot{\infty}_X$ converges to all elements of X^*, while in any other case the convergence behaviour of a filter $\mathcal{F}$ on X^* is determined by the convergence behaviour of the trace $i^{-1}(\mathcal{F})$ on X (note: $i^{-1}(\mathcal{F})$ exists iff $\mathcal{F} \neq \dot{\infty}_X$). Then (X^*, q^*) is the desired one–point extension of (X, q).

b) $\mathbf{KConv_s}$ is not extensional (cf. [38]).

5. Strong topological universes

5.1 Definitions. Let $\mathcal{C}$ be a topological construct. Consider the following *convenient properties*:

CP_1) $\mathcal{C}$ is cartesian closed.

CP_2) $\mathcal{C}$ is extensional.

CP_3) In $\mathcal{C}$ products of quotients are quotients.

Then $\mathcal{C}$ is called

1) *strongly cartesian closed* provided that $\mathcal{C}$ fulfills CP_1) and CP_3),
2) a *topological universe* provided that $\mathcal{C}$ fulfills CP_1) and CP_2),
3) a *strong topological universe* provided that $\mathcal{C}$ fulfills CP_1), CP_2) and CP_3).

5.2. Proposition. *Let $\mathcal{A}$ be a topological construct.*
1) If $\mathcal{B}$ is a bicoreflective subconstruct of $\mathcal{A}$ which is closed under formation of products in $\mathcal{A}$, then $\mathcal{B}$ fulfills CP_3) whenever $\mathcal{A}$ fulfills CP_3).
2) If $\mathcal{B}$ is a bireflective subconstruct of $\mathcal{A}$ which is closed under formation of quotients in $\mathcal{A}$, then $\mathcal{B}$ fulfills CP_3) whenever $\mathcal{A}$ fulfills CP_3).

5.3 Examples. 1) **Top** does not fulfill any of the above convenient properties (cf. 3.7.1) and 4.6.1)).

2) **Unif** fulfills CP_3) (cf. [23]), but it does not fulfill CP_1) and CP_2) (cf. 3.7.1) and 4.6.2)). The question whether **ULim** fulfills CP_3) is unsolved.

3) **SUConv** is a strong topological universe (cf. 3.7.3)a), 4.6.3) and [30;3.2.]).

4) **Fil** is a strong topological universe (cf. 3.7.4), 4.6.4) and note that according to 5.2., CP_3) follows from the fact that **Fil** is bicoreflective and bireflective in **SUConv**).

5) $\mathbf{KConv_s}$ is a strongly cartesian closed topological construct (cf. 3.7.6) and note that according to 5.2., CP_3) follows from the fact that $\mathbf{KConv_s}$ is bicoreflective in **SUConv** and closed under formation of products in **SUConv** by [31;3.13]), but it is not a strong topological universe (cf. 4.6.5)b)).

6) **Chy** is a cartesian closed topological construct (cf. 3.7.4)), but it fulfills neither CP_2) nor CP_3) (cf. [6]).

5.4. The deficiencies of **Top** and **Unif** lead to the following question:

What is the right framework for handling problems of a topological nature?

Obviously, it would be desirable to work in a strong topological universe in which convergence structures and uniform convergence structures are available. Furthermore, such a strong topological universe should be easily described by means of suitable axioms and should not be too big. The strong topological universe **SUConv** of semiuniform convergence spaces fulfills these criteria. Once having adopted *semiuniform convergence spaces as the right concept of space in topology*, it turns out that all classical results on topological and uniform spaces are a decisive part of the theory of semiuniform convergence spaces, because the study of semiuniform convergence spaces means exactly the study of **SUConv**–invariants, i.e. properties of semiuniform convergence spaces which are preserved by isomorphisms in **SUConv**. Thus, the study of subconstructs of **SUConv** (such as **Unif** and $\mathbf{Top_s}$ as well as all the other subconstructs according to 2.10.) is included. This point of view is summarized under the name

Convenient Topology.

6. Further Aspects

In the following we will mention some results which cannot be obtained by using topological or uniform spaces respectively.

6.1. Local compactness

6.1.1 Definitions. 1) A semiuniform convergence space $(X, \mathcal{J}_X)$ is called *compact* provided that for each ultrafilter $\mathcal{U}$ on X there is some $x \in X$ with $(\mathcal{U}, x) \in q_{\gamma_{\mathcal{J}_X}}$ (cf. 2.11.).

2) A subset A of a semiuniform convergence space $(X, \mathcal{J}_X)$ is called *compact* provided that $(A, \mathcal{J}_A)$ is compact, where $\mathcal{J}_A$ is the initial **SUConv**–structure on A with respect ot the inclusion map $i : A \to X$.

3) A semiuniform convergence space $(X, \mathcal{J}_X)$ is called *locally compact* provided that each $\mathcal{F} \in \mathcal{J}_X$ contains a compact subset of the product space $(X, \mathcal{J}_X) \times (X, \mathcal{J}_X)$.

6.1.2 Theorem (cf. [32;3.9.]). *The construct* **LC–SUConv** *of locally compact semiuniform convergence spaces (and uniformly continuous maps) is cartesian closed and topological.*

6.1.3 Theorem (cf. [32;3.12.]). *A semiuniform convergence space $(X, \mathcal{J}_X)$ is compactly generated, i.e. $\mathcal{J}_X$ is the final* **SUConv**–*structure with respect to the family $(j_i : (K_i, \mathcal{J}_{K_i}) \to (X, \mathcal{J}_X))$ of the inclusions of all compact subspaces of $(X, \mathcal{J}_X)$, if and only if it is locally compact.*

6.1.4 Remark. Concerning 6.1.2. and 6.1.3. the Hausdorff axiom (i.e. the uniqueness of filter convergence) is not assumed. As is well–known in the realm of topological spaces, the concepts 'compactly generated' and 'locally compact' do not coincide, and furthermore, (compact Hausdorff)–generated spaces form a cartesian closed subconstruct of **Top**, whereas (compact)–generated spaces do not form a cartesian closed subconstruct. Last but not least even (compact Hausdorff)–generated topological spaces have not been described by means of suitable axioms.

6.2 Local precompactness

6.2.1 Definitions. 1) A semiuniform convergence space $(X, \mathcal{J}_X)$ is called *precompact* (or *totally bounded*) provided that for each ultrafilter $\mathcal{U}$ on X, $\mathcal{U} \in \gamma_{\mathcal{J}_X}$ (cf. 2.11.).

2) A subset A of a semiuniform convergence space $(X, \mathcal{J}_X)$ is called *precompact* provided that A is precompact as a subspace.

3) A semiuniform space $(X, \mathcal{J}_X)$ is called *locally precompact* provided that each $\mathcal{F} \in \mathcal{J}_X$ contains a precompact subset of the product space $(X, \mathcal{J}_X) \times (X, \mathcal{J}_X)$.

6.2.2 Theorem (cf. [3;3.12]). *The construct* **LPC–SUConv** *of locally precompact semiuniform convergence spaces (and uniformly continuous maps) is a topological universe.*

6.2.3 Theorem (cf. [33;3.15]) *A semiuniform convergence space $(X\mathcal{J}_X)$ is precompactly generated, i.e. $\mathcal{J}_X$ is the final* **SUConv**–*structure with respect to the family $(j_i : (K_i, \mathcal{J}_{K_i}) \to (X, \mathcal{J}_X))$ of the inclusions of all precompact subspaces of $(X, \mathcal{J}_X)$, if and only if it is locally precompact.*

6.2.4 Remarks. 1) 6.2.3. is used (together with the cartesian closedness of **SUConv**) to characterize precompactness in the natural function spaces of **SUConv** (cf. [34] and [43]), a useful tool for deriving the classical Ascoli theorem.

2) It is not hard to derive from 6.2.2. that the construct **LPC–Fil** of locally precompact filter spaces (and Cauchy continuous maps) is a topological universe (cf. [33;4.1.]). This one has been used by H.L. Bentley and H. Herrlich [5] for proving Ascoli type theorems.

6.3. Local connectedness

6.3.1 Definitions. 1) Let $\mathcal{E}$ be a class of semiuniform convergence spaces. Then a semiuniform convergence space $(X, \mathcal{J}_X)$ is called *$\mathcal{E}$–connected* provided that each uniformly continuous map $f : (X, \mathcal{J}_X) \to (E, \mathcal{J}_E)$ is constant for each $(E, \mathcal{J}_E) \in \mathcal{E}$.

2) A semiuniform convergence space $(X, \mathcal{J}_X)$ is called *locally $\mathcal{E}$–connected* provided that for each $\mathcal{F} \in \mathcal{J}_X$, there is some subfilter $\mathcal{G} \in \mathcal{J}_X$ together with a filter base $\mathcal{B}$ for $\mathcal{G}$ consisting of $\mathcal{E}$–connected subsets of $(X, \mathcal{J}_X) \times (X, \mathcal{J}_X)$.

6.3.2 Remarks. 1) If $\mathcal{E} = \{(\{0,1\}, \{\dot{0} \times \dot{0}, \dot{1} \times \dot{1}\})\}$, then $\mathcal{E}$–connectedness means connectedness in the usual sense. If there $\mathcal{E} = \{D_2^\Delta\}$, where D_2^Δ denotes the two–point discrete uniform space, then $\mathcal{E}$–connectedness means uniform connectedness, e.g. a metric space (X, d) (considered as a uniform space) is uniformly connected iff it is *Cantor–connected* (i.e. for each $\varepsilon > 0$ and any pair of points $x, y \in X$ there is a finite sequence $x_1, \ldots, x_n$ of points of X with $x_1 = x, y_n = y$ and $d(x_i, x_{i+1}) < \varepsilon$ for each $i \in \{1, \ldots, n-1\}$. (cf. G. Cantor [7;p. 575]). Obviously, the metric space $\mathbb{Q}$ of rational numbers is Cantor–connected.

2) *The theory of $\mathcal{E}$–connectedness (resp. $\mathcal{E}$–disconnectedness) in the realm of semiuniform convergence spaces profits from the fact that* **SUConv** *is extensional* (cf. [31]).

6.3.3 Theorem (cf. [31;6.3.6]). *The construct* $\mathbf{LConv}_{\mathcal{E}}$ *of locally $\mathcal{E}$–connected semiuniform convergence spaces (and uniformly continuous maps) is cartesian closed and topological.*

6.4 Completion of uniform spaces via natural function spaces

As is well–known every metric space $\mathbf{X}$ has a completion which can be obtained by means of a certain function space of real–valued functions on $\mathbf{X}$ (cf. e.g. [21;3.6.3 and 3.6.4.]). In the realm of uniform spaces a corresponding procedure is unknown. But if **Unif** is considered to be embedded into **ULim**, a completion for separated uniform spaces, which is isomorphic to the usual Hausdorff completion for separated uniform spaces, can be obtained via the natural function spaces in **ULim**. This has been demonstrated by Gazik, Kent and Richardson [16]. Since the power objects in **ULim** are formed as in **SUConv**, their results remain valid in the realm of semiuniform convergence spaces.

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