Constraints on Derivations

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Abstract

We study the effect of various constraints on derivations: *Cyclicity*, *Freeze*, and *Shortest Steps*. It will be shown that given a surface structure, one can effectively determine in polynomial time an ordering on the chain links such that any linear ordering extending it gives rise to a derivation. The length of the steps can be computed as well. Using this, it is shown that up to inessential variations *Freeze* and *Shortest Steps* select only one derivation as the right one, while *Cyclicity* leaves (in the worst case) exponentially many choices. Moreover, the combined effect of *Shortest Steps* and *Cyclicity* is determined, once again leading to an essentially unique solution.

1 Introduction

In [5] we have proposed a theory of chains in which it is demonstrated that there exist three concurrent types of representations, *Copy Chain Structures* (*CCSs*), *Trace Chain Structures* (*TCSs*) and *Multidominance Structures* (*MDSs*). The first arise from copy movement as used in the MP, the second from copy and delete as used in GB (giving rise to traces), and the third arising from linking as used in Linking Grammars. Moreover, there is a bijective correspondence between derivations of structures of the respective types. Thus, viewed from the standpoint of derivations, the representations are equivalent. Moreover, there is a biunique correspondence between trace chain structures and MDSs, so that the latter two representations are equivalent in all respects.

Motivations for this investigation as well as references may be found in [5]. In this paper we will concentrate on the possible derivations that can reach a given structure. This will help to elucidate the role that derivations play in syntactic theory, in particular in answering theoretical questions such as whether there is a theoretical need for derivations at all. The main question we ask is the following

> *Given a syntactic structure S, is there a derivation of this structure meeting certain given derivational constraints?*

Partial answers have been given in [5]. However, the paper contains a mistake concerning MDSs, which we shall correct here. Evidently, given a constraint γ and a structure S, we can simply enumerate all derivations and see whether they satisfy γ. What we are aiming for is therefore some answer to this question that does not require checking all derivations. Moreover, as many constraints require the inspection of *all* derivations, similar considerations concerning the need of investigating the entire set arise. These considerations turn on the issue of locality, whether or not the possibility of a step can be checked prior to its execution. Constraints that have been proposed are among others: *cyclicity*, *freeze*, *shortest steps*, *minimum number of steps*. Additionally, there are principles concerning a single step: the antecedent must be local to the trace. The best know of such principles is of course subjacency. Freeze requires that no trace be unbound at any step of the derivation. There is an antagonism between those principles that require steps to be short (locality, shortest steps) and those that require steps to be long (cyclicity, freeze). The number of steps turns out to be an invariant. It is the same in all derivations.

In [5] copying is defined as an operation that encodes an explicit link between the upper and the lower copy. Therefore copying preserves more information about the derivation than does linking. In fact, a complete map of all possible derivations can be given by means of an ordering between the copying steps, which means a reduction in complexity from exponential to at most quadratic in size. Thus, copying results in structures that carry all necessary information on their derivation. This is not so with the other two: given a trace chain structure, there exist several derivations which differ fundamentally with respect to important constraints on derivations. In particular, the structures do not immediately reveal violations of locality. In a remnant movement structure, the distance antecedent–trace may become evident only after reconstruction:

$$[\alpha_1 \ \ldots [\beta \ t_1]_2 \ldots t_2]$$

Suppose that α has been extracted before $[\beta \ t_1]$. The distance between α and its trace is seen only when we undo the second step:

$$[\alpha_1 \ \ldots e \ldots [\beta \ t_1]]$$

Moreover, as we potentially need to check exponentially many different derivations, the computational load may be very high. We shall show that the situation is not so bad after all. The effect of removing a single link on the minimum distance between the elements of the structure can be established almost independently of the derivation. Further, one can tabulate the dependency between the various derivational steps. After that, the space of possible derivations can be compactly represented. In particular, checking whether there exists a shortest steps derivation satisfying locality (for any regular locality measure) is straightforward.

2 Trees and Command Relations

Before we begin, let us say a few words on the general setup. In order to simplify the exposition, we have chosen to assume the following: (a) movement is substitution, never adjunction, (b) there is no covert or LF–movement, (c) there is no left–to–right ordering on the structures, and (d) trees are binary branching. Neither of the simplifications is necessary, but without them the exposition would become rather imperspicuous and the important insights will be lost. The interested reader is referred to [4] for the complications arising from adjunction and [5] for the complications arising from ordering and covert movement.

Definition 1 *A **tree** is a pair $\langle T, \prec \rangle$ such that the following holds.*

1. *T is finite.*
2. *$\prec \subseteq T^2$.*
3. *$< := \prec^+$ is irreflexive.*
4. *There is an r such that for all $x \neq r$: $x < r$.*

5. *For every $x \neq r$ there is exactly one y such that $x \prec y$.*

*An A–**tree** is a triple $\langle T, <, \ell \rangle$ such that $\langle T, < \rangle$ is a tree and $\ell : T \to A$ a function, the so called **labelling function**.*

Definition 2 *x c–commands y if for all $z > x$: $z \geq y$. x **ac–commands** y if (a) x and y are incomparable, (b) x c–commands y, (c) y does not c–command x.*

In a tree, a lower set $\downarrow x := \{y : y \leq x\}$ as well as the (A–)tree induced on it, are called **constituents** of that tree. x is called the **generator** of $\downarrow x$. We say that $\downarrow x$ **covers** $\downarrow y$ if $\downarrow y \subseteq \downarrow x$ iff $y \leq x$. Further, $\downarrow x$ c–commands $\downarrow x$ if and only if x c–commands y (and likewise for ac–command and analogous notions). We use Gothic letters ($\mathfrak{C}$, $\mathfrak{D}$ and so on) as variables for constituents.

Our main target of investigation are notions of locality. As we have outlined in earlier work, locality is a central tool in transformational grammar and can be adequately captured in terms of command relations. Given two nodes x, y, we pick the least node z dominating x and y and study the paths $[x, z]$ and $[y, z]$. For ease of presentation we exclude adjunction from our considerations. Then we know that one of the two intervals is trivial, consisting of at most two nodes. Locality is then reduced to studying admissible paths leading from one node to another. We assume that if y is local to x and $y \leq y' \leq z$ then also y' is local to x. Barker and Pullum have surveyed the notions of locality used in the literature and given a definition of command relation in [1]. Using this, Kracht [3] developed a mathematical theory of command relations that we shall use here.

Definition 3 *Let $\langle T, < \rangle$ be a tree and $R \subseteq T^2$ a relation. R is called a **command relation** if there is a function $f_R : T \to T$ such that (1) – (3) hold. R is called a **monotone command relation** if in addition it satisfies (4), and it is called **tight** if it satisfies (1) – (5).*

1. $R_x := \{y : x \; R \; x\} = \downarrow f_R(x)$.

2. $x < f_R(x)$ *for all* $x < r$.

3. $f_R(r) = r$.

4. *If* $x \leq y$ *then* $f_R(x) \leq f_R(y)$.

5. *If* $x < f_R(x)$ *then* $f_R(x) \leq f_R(y)$.

The first class of CRs that we shall study is the class of tight command relations. Let $\mathfrak{T}$ be a tree and $P \subseteq T$. We say, x **P–commands** y if for every $z > x$ with $z \in P$ we have $z \geq y$. We denote the relation of P–command by $K(P)$. If we choose $P = T$ we exactly get c–command. The following theorem is from [3].

Proposition 4 *Let R be a binary relation on the tree $\langle T, < \rangle$. R is a tight command relation if and only if $R = K(P)$ for some $P \subseteq T$.*

What we are really aiming at are not particular relations but rather schemes of relations. These are uniformly defined on all A–trees.

Definition 5 *A **relation scheme over** A–**trees** is a function R that assigns to every A–tree $\mathfrak{T} = \langle T, \prec, \ell \rangle$ a binary relation $R(\mathfrak{T})$. The **scheme of** B–**command** for $B \subseteq A$ is the scheme which assigns to $\mathfrak{T}$ the relation of $\ell^{-1}[B]$–command. Any such scheme is called a **tight command scheme**.*

Let $\mathfrak{T}$ be a tree. We denote by $\mathrm{MCr}(\mathfrak{T})$ the set of monotone command relations on $\mathfrak{T}$. This set is closed under intersection, union and relation compositon. We even have

$$\begin{array}{rcl} f_{R\cup S}(x) & = & \max\{f_R(x), f_S(x)\}, \\ f_{R\cap S}(x) & = & \min\{f_R(x), f_S(x)\}, \\ f_{R\circ S}(x) & = & (f_S \circ f_R)(x). \end{array}$$

For we have $(R \cup S)_x = R_x \cup S_x = \downarrow f_R(x) \cup \downarrow f_S(x) = \downarrow(\max\{f_R(x), f_S(x)\})$, and likewise for intersection. For relation composition we need the monotony. For if $x\, R \circ S\, y$ we can conclude that $x\, R\, f_R(x)$ and $f_R(x)\, S\, y$. Hence $x\, R \circ S\, y$ if and only if $y \leq f_S(f_R(x))$, from which the claim now follows. Now we set

$$\mathfrak{MCr}(\mathfrak{T}) = \langle \mathrm{MCr}(\mathfrak{T}), \cap, \cup, \circ \rangle \ .$$

$\mathfrak{MCr}(\mathfrak{T})$ is a distributive lattice with respect to $\cap$ and $\cup$. What is more, there are additional laws of distribution concerning relation composition.

Proposition 6 *Let R, S, T be from $\mathrm{MCr}(\mathfrak{T})$. Then*

1. $R \circ (S \cap T) = (R \circ S) \cap (R \circ T), \qquad (S \cap T) \circ R = (S \circ R) \cap (T \circ R).$
2. $R \circ (S \cup T) = (R \circ S) \cup (R \circ T), \qquad (S \cup T) \circ R = (S \circ R) \cup (T \circ R).$

Likewise for $\cup$.

Again, this is from [3].

Definition 7 *Let $\mathfrak{T}$ be a tree, $R \in \mathrm{MCr}(\mathfrak{T})$. R is called* ***generated*** *if R can be produced from tight command relations by means of $\cap$, $\cup$ and $\circ$. R is called* ***chain like*** *if it can be generated from tight relations with $\circ$ alone.*

Theorem 8 *R is generated if and only if R is an intersection of chain line command relations.*

The whole construction can be lifted to relation schemes.

Definition 9 *Let R be a relation scheme on A–trees. R is called a* ***(definable) command relation (scheme)*** *if it can be produced from tight command schemes by means of composition, union and intersection.*

To see that the theory is nontrivial we give the following illustration. A particular role is played in grammar by subjacency. The antecedent of a trace must be 1–subjacent to a trace. As is argued in Kracht [4] on the basis of Chomsky [2], this relation is exactly

$$K(\mathrm{IP}) \circ K(\mathrm{CP}) .$$

Definition 10 *Let $\mathfrak{T} = \langle T, \prec, \ell \rangle$ be an A–tree and $U \subseteq T$. Then put $x \prec_U y$ if y is the smallest node of U such that $x < y$. Then $\mathfrak{T} \upharpoonright U := \langle U, \prec_U, \ell \upharpoonright U \rangle$ is the* ***subtree induced by*** *U.*

If R is a command scheme and $\mathfrak{T}$ a tree, we say that x R–**commands** y **in** $\mathfrak{T}$ if $x \; R(\mathfrak{T}) \; y$.

Proposition 11 (Subset Property) *Let R be a definable command relation and $\mathfrak{T}$ an A–tree. Further, let $U \subseteq T$ and $x, y \in U$. Then if x R–commands y in $\mathfrak{T}$, x R–commands y in $\mathfrak{T} \upharpoonright U$ as well.*

Proof. By induction on R. Suppose that R is tight, say it is B–command for some $B \subseteq A$. Then x B–commands y in $\mathfrak{T}$ if for the first node z with label in B which is $> x$ we have $z \geq y$. (If such a node does not exist, x B–commands y anyway, and then this is so also in $\mathfrak{T} \upharpoonright U$.) Now take the least node z' in U such that $z' > x$ and $\ell(z') \in B$. If it does not exist, x B–commands y in $\mathfrak{T} \upharpoonright U$. If it does, then clearly $z' \geq z$. So, also in this case x B–commands y. The steps for union and intersection are straightforward. Q. E. D.

3 Copy Chain Structures

Before we begin, we shall make a note of comparison with [5]. In the latter we have been dealing also with LF–movement. The complications are left aside here. Additionally, the terminology is implified as well. What appears in [5] as a *pre–chain*, a *preCCS* or a *preMDS* is now called a *chain*, a *CCS* or an *MDS*. This makes the presentation more perspicuous.

Definition 12 *Let $\mathfrak{T}$ be a tree. A* ***chain*** *is a set of constituents of $\mathfrak{T}$ which is linearly ordered by ac–command. The highest member of the chain (with respect to ac–command) is called its* ***head****, the lowest is called its* ***foot****. A* ***chain***** *is a pair $\langle \Delta, \Phi \rangle$ such that Δ is a chain and $\Phi = \{\varphi_{\mathfrak{C},\mathfrak{D}} : \mathfrak{C}, \mathfrak{D} \in \Delta\}$ a family of isomorphisms such that*

1. $\varphi_{\mathfrak{C},\mathfrak{C}} = 1_{\mathfrak{C}}$ *for all* $\mathfrak{C} \in \Delta$.
2. $\varphi_{\mathfrak{C},\mathfrak{D}} \circ \varphi_{\mathfrak{D},\mathfrak{F}} = \varphi_{\mathfrak{C},\mathfrak{F}}$ *for all* $\mathfrak{C}, \mathfrak{D}, \mathfrak{F} \in \Delta$.

If $\mathfrak{D}$ immediately ac–commands $\mathfrak{C}$ in Δ we call the triple $\langle \mathfrak{D}, \varphi_{\mathfrak{C},\mathfrak{D}}, \mathfrak{C} \rangle$ a ***link*** *and the map $\varphi_{\mathfrak{C},\mathfrak{D}}$ a* ***link map****.*

Evidently, a chain is uniquely characterized by its links. In ordinary theory the isomorphisms are needless since the trees are assumed to be ordered anyway. However, in transformational theory it is often assumed that trees are unordered in syntax, whence the isomorphisms must be explicitly given (see [5]). If we use A–trees, then the definitions remain as they are. It follows that if $\mathfrak{C} = \downarrow x$ and $\mathfrak{C} = \downarrow y$ are in the same chain, then $\ell(x) = \ell(y)$.

In what is to follow, we shall be interested in the distance covered by an element that is moved. This distance is measured on the basis of the *path of link*.

Definition 13 *Let $\varphi : \downarrow x \to \downarrow y$ be a link map and $z \succ y$. The open interval $]x, z[:= \{u : x < u < z\}$ is called the* ***path set of*** *φ. Moreover, let $]x, z[= \{u_i : i < n\}$ be an enumeration such that $u_i \prec u_{i+1}$ for all $i < n - 1$. Then the sequence $\langle \ell(u_i) : i < n \rangle$ is called the* ***path expression of*** *φ and denoted by $\Pi(\varphi)$.*

The following is easily checked.

Proposition 14 *Suppose that $\varphi : \downarrow x \to \downarrow y$ is a link map. Then x B–commands y iff $\Pi(\varphi)$ contains no occurrence of an element of B.*

Definition 15 *A **copy chain structure (CCS)** is a pair $\langle\mathfrak{T}, K\rangle$ (which we often simply write K) where $\mathfrak{T}$ is a tree and K a set of chains* of $\mathfrak{T}$ such that the following is satisfied.*

Chain Existence *Every constituent is in a chain*.*

Chain Uniqueness *Every constituent is in at most one chain*.*

Liberation *If a member $\mathfrak{C}$ of a chain* Γ covers two distinct members of some other chain*, then $\mathfrak{C}$ is the foot of Γ.*

No Recycling *Every link must be orbital.*

We still have to define what it means that a link is *orbital*.

Definition 16 *Let $\langle\mathfrak{T}, K\rangle$ be a CCS, $\Gamma = \langle\Delta, \Phi\rangle \in K$. Put $x \approx_\Gamma y$ iff $x = \varphi(y)$ for some $\varphi \in \Phi$. This is an equivalence relation. Further, let $\approx_K$ be the smallest equivalence relation containing the $\approx_\Gamma$, $\Gamma \in K$.*

First, we define the depth of an element as follows. For the root r we put $t(r) := 0$, and else if $x \prec y$ we put $t(x) := t(y) + 1$. Let x be an element. Then there is a unique element x_r such that (a) $x_r \approx x$ and (b) $t(y) < t(x_r)$ for all $y \approx x_r$. This is called the **root of** x.

For given x there is at most one link map φ such that $x \in dom(\varphi)$. If we apply this to the root x_r we can find for every $y \approx x_r$ a composition of link maps χ such that $\chi(x_r) = y$. This composition is unique for given y and is called the **natural composition of** y. The following is immediate if we observe that $x \approx_K y$ is equivalent with $x_r = y_r$.

Proposition 17 *In a CCS, $x \approx_K y$ iff there are natural compositions φ and χ such that $y = \varphi \circ \chi^{-1}(x)$.*

The **root line of** x is the set of all $u \approx x$ which c–command x_r. The **peak of** x, x_p, is the element in the root line of x of least depth. The **peak map of** x, π_x, is the natural composition of x_p. Finally, the zenith of x as well as the zenith map is defined inductively as follows.

Definition 18 *Let K be a CCS over $\mathfrak{T}$. Let r be the root of $\mathfrak{T}$. The zenith of r, r_ζ,is r and the zenith map, ζ_r, is the identity. For other elements, the **zenith of** x, x_ζ, is $\zeta_y(x_p)$, where y is the mother of x_p. The **zenith map of** x, ζ_x, is $\zeta_y \circ \pi_x$.*

In order to know where the surface equivalent of x is we have to look at x_ζ. The zenith of x_ζ contains the orbital maps for x.

Definition 19 *A link map is **orbital** if it is contained in the natural decomposition of ζ_x for some x.*

This clarifies the formal character of the postulates. What it means in content, however, is that a link map may not move a constituent whose root is empty. It is checked that *No Recycling* is exactly the postulate that ensures this.

4 Multidominance Structures

Given a CCS, we can construct a structure as follows. Put

$$\begin{array}{lcl} [x]_K & := & \{y : x \approx_K y\} \\ M(T) & := & \{[x]_K : x \in T\} \\ \prec_K & := & \{\langle [x]_K, [y]_K \rangle : \langle x, y \rangle \in K\} \\ M(K) & := & \langle M(T), \prec_K \rangle \end{array}$$

Notice that for A–trees, if $x \approx_K y$ implies $\ell(x) = \ell(y)$. $M(K)$ is a multidominance structure, which is in general defined as follows.

Definition 20 *A **multidominance structure (MDS)** is a pair $\langle M, \prec \rangle$ such that the following holds.*

1. *$< := \prec^+$ is irreflexive.*
2. *There is a $\rho \in M$ such that for all $\alpha \neq \rho$: $\alpha < \rho$.*
3. *For all α the set $M(\alpha) := \{\beta : \langle \alpha, \beta \rangle \in \prec\}$ is linearly ordered by $<$.*

To facilitate reading this paper we use α, β, γ to range over points of an MDS, while x, y and z range over points of a CCS. The following is shown in [5].

Theorem 21 *For every MDS $\mathfrak{M}$ there exists a CCS K such that $M(K) \cong \mathfrak{M}$. There may exist nonisomorphic CCS K, K' such that $M(K) \cong M(K')$.*

Now, the literature in transformational grammar actually uses a third kind of structure, which is similar to a CCS, only that chains consist of an overt element (the head of the chain, since we do not deal with LF–movement, for simplicity's sake), and several empty elements, called *traces*. Such structures are called **trace chain structures** (**TCSs**). It turns out that there is a bi-unique correspondence between TCSs and MDSs. However, from a technical viewpoint, MDSs are better behaved than TCSs (see [5] for a discussion). Hence we shall focus here on MDSs instead. We shall give the construction here for completeness' sake.

Definition 22 *A **trace** is a one node tree with root label t. A **trace chain** is a chain such that all but the head of the chain are traces.*

Now, trace chain structures are those structures that arise from a CCS by removing every node x such that $x < y$ for some y scuh that $\downarrow y$ is not the head of a chain. It means that empty material is not layered. This elimination will actually remove certain constituents from chains. So a direct definition is a tricky to obtain.

Given a trace chain Γ, put $x \approx_\Gamma y$ iff either $x = y$ or x and y are the generators of some constituents of Γ. Given a set K of trace chains, $\approx_K$ is the least equivalence relation containing all $\approx_\Gamma$, $\Gamma \in K$. The postulate of *No Recycling* boils down to the requirement on trace chains that the head of a chain may not be a trace (so it must be a proper constituent). Now set once again:

$$\begin{array}{lcl} [x]_K & := & \{y : x \approx_K y\} \\ M(T) & := & \{[x]_K : x \in T\} \\ \prec_K & := & \{\langle [x]_K, [y]_K \rangle : \langle x, y \rangle \in K\} \\ M(K) & := & \langle M(T), \prec_K \rangle \end{array}$$

This is an MDS.

One main interest is in the construction of a K such that $M(K) \cong \mathfrak{M}$. In fact, while [5] focusses on the derivations of a given CCS, here we are looking at derivations of a given MDS, in particular with respect to locality conditions. Since any derivation of the same CCS satisfies the same locality conditions, it is the different CCSs that have the same MDS which are of interest when it comes to locality.

A pair $\lambda = \langle \alpha, \beta \rangle$ is called a **link** if it is in $\prec$. Two links $\lambda = \langle \alpha, y \rangle$ and $\lambda' = \langle \alpha', \beta' \rangle$ can be **composed** iff $\alpha' = \beta$, and the result of the composition, denoted by $\lambda \circ \lambda'$ equals $\langle \alpha, \beta' \rangle$. $\circ$ is a partial partial function from $\prec^2$ to $\prec$.

Lemma 23 *There is a polynomial algorithm computing the relation $<$ running in at most $O(n^3)$ time.*

Assume that $\prec\ = \{\mu(i) : i < \xi\}$. A **path of length** k is a sequence $\Pi = \langle \lambda_i : i < k \rangle$, $\lambda_i = \langle \alpha_i, \beta_i \rangle$ of links such that for all $i < k-1$ λ_i is composable with λ_{i+1}. The **begin point of** Π is α_0, and β_{k-1} is the **end point of** Π. An alternative definition is a sequence $\langle \alpha_i : i < k+1 \rangle$ such that for all $i < k$ $\langle \alpha_i, \alpha_{i+1} \rangle$ is a link. Given α and β, we write $d(\alpha, \beta)$ for the minimum of all lengths of paths from α to β. A path of length $d(\alpha, \beta)$ is called a **shortest path from** α **to** β. Shortest paths are not unique. For example, let $\mathfrak{M} = \langle \{0, 1, 2, 3\}, \prec \rangle$ with $0 \prec 1 \prec 2 \prec 3$, $0 \prec 2$ and $1 \prec 3$. Then $\langle 0, 1, 3 \rangle$ and $\langle 0, 2, 3 \rangle$ are both shortest paths from 0 to 3.

Definition 24 *Let $\mathfrak{M} = \langle M, \prec \rangle$ be an MDS. Let $T(\mathfrak{M})$ denote the set of all paths ending at the root. Further, we put $\Pi \prec_T \Pi'$ if $\Pi = x; \Pi$ for some $x \in M$ and $\Pi \approx \Pi'$ if Π and Π' begin at the same point.*

So, $\Pi \approx \Pi'$ iff there is an α and Σ, Σ', arbitrary, such that $\Pi = \alpha; \Pi$ and $\Pi' = \alpha; \Sigma'$.

Lemma 25 *Then $\langle T(\mathfrak{M}), \prec_T \rangle$ is a tree. Moreover, for a CCS $\langle \mathfrak{T}, K \rangle$, $\mathfrak{T}$ is isomorphic to $\langle T(M(K)), \prec_T \rangle$.*

Proof. That $\langle T(\mathfrak{M}), \prec_T \rangle$ is a tree is immediately verified. The isomorphism is constructed in stages. If r is the root of $\mathfrak{T}$, set $\tau_K(r) := [r]_K$. If τ_K is defined on y and $x \prec y$ then put $\tau_K(x) := [x]_K; \tau_K(y)$. Several things need to be seen. (i) τ_K is injective. For this it is enough to show that if $x, x' \prec y$ and $[x]_K = [x']_K$ then $x = x'$. Now let $x, x' \prec y$ and $[x]_K = [x']_K$. Then there exist natural compositions φ and χ such that $x = \varphi \circ \chi^{-1}(x')$. Equivalently, $x = \varphi(x_r)$, $x' = \chi(x_r)$. By induction on the length of φ it is shown that φ and χ are identical. (ii) τ_K is surjective. This is shown by induction. If $\Pi = [r]_K$, the case is clear. If $\Pi = [x]_K; \Pi'$, there is a y such that $\Pi' = [y]_K; \Pi''$. By induction hypothesis there is a y' such that $\Pi' = \tau_K(y')$. Then there is a $x' \prec_K y'$ such that $x' \approx_K x$. It follows that $\Pi = [x']_K; \Pi'$ so that $\Pi = \tau_K(x')$. (ii) $x \prec_K y$ iff $\tau_K(x) = \tau_K(y)$. This is shown for example by induction on the depth of y. Q. E. D.

Lemma 26 *Let $\langle \mathfrak{T}, K \rangle$ be a CCS. Then $x \approx_K y$ iff $\tau_K(x) \approx \tau_K(y)$.*

Proof. $x \approx_K y$ iff $[x]_K = [y]_K$ iff $\tau_K(x)$ begins with the same element as $\tau_K(y)$ (by construction of τ_K, $\tau_K(x)$ begins with $[x]_K$). Q. E. D.

So, given an MDS we can actually recover the tree structure, and also the equivalence relation induced by the chains in their totality. This does not mean, however, that we can recover the individual chains. Nevertheless, we can recover for each x the peak of x (as it is the highest member of the root line) and the zenith of each element (which is defined by recursion over the tree structure and the peaks). This allows to recover the surface structure.

Definition 27 *Let $\mathfrak{M} = \langle M, \prec\rangle$ be an MDS. A link $\lambda = \langle\alpha, \beta\rangle$ is **maximal** if β is the largest element of $M(\alpha)$ with respect to $<$. $\mu(\mathfrak{M})$ denotes the set of maximal links of $\mathfrak{M}$. Further, a path Π is a **surface path** if $\Pi \in \mu(\mathfrak{M})^*$, that is, if it consists entirely of maximal links. For each $\alpha \in M$ the **surface path** of α is the unique surface path beginning with α. It is denoted by $\sigma_{\mathfrak{M}}(\alpha)$.*

Shortest paths need not be unique. For example, take $M := \{0, 1, 2, 3\}$ and $\prec = \{\langle 0, 1\rangle, \langle 1, 2\rangle, \langle 2, 3\rangle, \langle 0, 2\rangle, \langle 1, 3\rangle\}$. Then $\langle 0, 1, 3\rangle$ and $\langle 0, 2, 3\rangle$ are shortest paths from 0 to 3.

Definition 28 *Let $\lambda = \langle\alpha, \beta\rangle$ be a link. Define $\mu(\alpha; \beta)$ as follows. (a) If $\langle\alpha, \gamma\rangle \in \mu(\mathfrak{M})$ and $\gamma < \beta$ then $\mu(\alpha; \beta) := \alpha; M(\gamma; \beta)$. (b) If $\langle\alpha, \beta\rangle \in \mu(\mathfrak{M})$, put $\mu(\alpha; \beta) := \alpha; \beta$. Otherwise, $\mu(\alpha; \beta)$ is undefined.*

5 Derivations of CCSs and MDSs

In [5] we have defined a blocking order on the link maps. Subsequently, we have shown that given any linear order on the link maps that extends the blocking order, there is a derivation that inserts the link maps in just that given order. This gives a complete overview over the possibly derivations of a given CCS. In this section, we do something similar for MDSs. However, here matters are more delicate. In a CCS, the paths defined by the link maps are fixed and do not vary between the derivations. Hence, the question of shortest links does not arise. It only arises when we look at MDSs. Although [5] has given algorithms for constructing CCSs form a given MDS, it was not shown how to systematically enumerate all possible MDSs and morever determine in an efficient way the length of the arising links. This is what we shall do now.

We define the notion of a 1–step extension for a chain and use this to define a step in a derivation of CCSs.

Definition 29 *Let Δ and Σ be chains. Δ is a **1–step extension** of Σ if $\Delta = \{\mathfrak{C}_i : i < n+1\}$, $\Sigma = \{\mathfrak{C}_i : i < n\}$ for a certain $n \in \omega$ and constituents $\mathfrak{C}_i$, and $\mathfrak{C}_n$ ac–commands all $\mathfrak{C}_i$, $i < n$.*

Analogously for chains*. So, a 1–step extension is a chain with one more chain link. For chains* we also have to define a new chain link* $\langle \mathfrak{C}_n, \varphi_{n-1,n}, \mathfrak{C}_{n-1}\rangle$, with the new isomorphism $\varphi_{n-1,n}$. The full set of isomorphisms of the new chain* is obtained by closing the old set plus $\varphi_{n-1,n}$ under composition and inverse. (For the purpose of the next definition, let $m(\Gamma)$ be the set of nodes contained in a constituent of Γ.)

Definition 30 *Let $\mathbb{K}_1 = \langle \mathfrak{T}_1, K_1\rangle$ and $\mathbb{K}_2 = \langle \mathfrak{T}_2, K_2\rangle$ be CCSs. $\mathbb{K}_2$ is obtained from $\mathbb{K}_1$ by **(1–step) copy–movement** if the following holds.*

1. *$\mathfrak{T}_1 \subseteq \mathfrak{T}_2$.*
2. *$\mathfrak{C} := \mathfrak{T}_2 - \mathfrak{T}_1$ is a constituent of $\mathfrak{T}_2$.*
3. *$\mathfrak{C}$ is in zenith position in $\mathbb{K}_2$.*
4. *Every chain of $\mathbb{K}_1$ is the residue of a chain of $\mathbb{K}_2$.*
5. *There is exactly one nontrivial chain Γ such that $m(\Gamma) \cap \mathfrak{T}_1 \neq \varnothing$ and $m(\Gamma) \cap \mathfrak{C} \neq \varnothing$. Moreover:*
   (a) *$m(\Gamma) \cap \mathfrak{C} = \mathfrak{C}$.*
   (b) *$\mathfrak{C}$ is the highest element of Γ.*
   (c) *For the second highest element $\mathfrak{D}$ of Γ: $\mathfrak{D}$ is in zenith position of $\mathbb{K}_1$.*

*$\mathbb{L}$ is obtained from $\mathbb{K}$ by **copy–movement** iff there exists a sequence $\mathbb{K}_i$, $i < n+1$, such that $\mathbb{K} = \mathbb{K}_0$, $\mathbb{L} = \mathbb{K}_n$ and $\mathbb{K}_{i+1}$ is obtained from $\mathbb{K}_i$ by 1–step copy–movement. The sequence $\langle \mathbb{K}_i : i < n+1\rangle$ is also called a **derivation** of $\mathbb{K}$. This derivation has **length** n.*

Definition 31 *A CCS is a **tree** if no chain has more than one member.*

Theorem 32 *For every CCS $\mathbb{K}$ there is a tree $\mathbb{K}_0$ and a derivation of $\mathbb{K}$ from $\mathbb{K}_0$.*

In [5] the following definition is given. Say that β is **not derived** if either β is the root or $M(\beta) = \{\gamma\}$, where γ is not derived. (So, above β the relation $<$ is linear.) The following is a corrected version of Definition 58 of [5].

Definition 33 *Let $\mathfrak{M} = \langle M, \prec_M\rangle$ and $\mathfrak{N} = \langle N, \prec_N\rangle$ be MDSs. $\mathfrak{N}$ is a **1–step link extension** of $\mathfrak{M}$ if $N = M$ and $\prec_N = \prec_M \cup\{\langle\alpha, \beta\rangle\}$, where $\langle\alpha, \beta\rangle \notin \prec_M$ and β is not derived and such that there is no link $\langle\alpha', \beta'\rangle$ such that $\alpha \leq \alpha' < \beta < \beta'$. $\mathfrak{N}$ is a **link extension** of $\mathfrak{M}$ if there exists a finite sequence $\langle\mathfrak{N}_i : i < n + 1\rangle$ such that $\mathfrak{N}_0 = \mathfrak{M}$, $\mathfrak{N}_n = \mathfrak{N}$ and $\mathfrak{N}_{i+1}$ is a 1–step link extension of $\mathfrak{N}_i$ for $i < n$.*

The following is a correct version of Theorem 59 of [5].

Theorem 34 *Every MDS is a link extension of a tree. More precisely, the following holds.*

1. *If L is isomorphic to a 1–step extension of K then $M(L)$ is isomorphic to a 1–step link extension of $M(K)$.*
2. *If $\mathfrak{M} = M(K)$ and $\mathfrak{N}$ is a 1–step link extension of $\mathfrak{M}$ then there exists a 1–step extension L of K such that $M(L) \cong \mathfrak{N}$. L is unique up to isomorphism.*

Proof. The first claim follows from the other claims by the fact that every CCS is derived from a tree. The proof that a one step extension of K gives rise to a link extension of $\mathfrak{M}$ and conversely remains to be given. Let $\langle\mathfrak{T}, K\rangle$ be a CCS. So, let $\langle\mathfrak{T}, K\rangle$ be a CCS and $\mathfrak{M}$ the corresponding MDS. We identify $\mathfrak{T}$ with $T(\mathfrak{M})$. Let us make a 1–step extension by adding a constituent. It is shown in [5] that this corresponds to the addition of a nonderived link $\langle\alpha, \beta\rangle$. We wish to establish further that there is no link $\langle\alpha', \beta'\rangle$ in $\mathfrak{M}$ such that $\alpha \leq \alpha' < \beta < \beta'$. (Case 1.) $\alpha = \alpha'$. Here, we have a link $\langle\alpha, \beta'\rangle$ in $\mathfrak{M}$ such that $\beta < \beta'$. This means, however, the following. Since β is underived, so is now β', and there are unique paths Π and Π' starting at β and β', respectively. They are the surface paths of β and β', respectively, and Π' is a suffix of Π. The surface path of α is therefore $\alpha; \Pi'$ in K and $\alpha; \Pi$ in L, which is a longer sequence. So, $\alpha; \Pi$ does not c–command $\alpha; \Pi'$. However, $\alpha; \Pi$ is the generator of the moved constituent. Contradiction to the definition of a chain. (Case 2.) $\alpha < \alpha'$. Similar to the first case. Now, we show the converse: let $\mathfrak{M}$ correspond to K and let $\mathfrak{N}$ be a link extension of K. We show that there is a 1–step extension L of K such that $M(L) \cong \mathfrak{N}$.

We put $L = \langle \mathfrak{U}, L \rangle$, $\mathfrak{U} := T(\mathfrak{N})$. Now, the link extension adds a link $\langle \alpha, \beta \rangle$, hence (1.) and (2.) are clear. (3.) β is underived. This means that there is a unique sequence Π starting in β. This means that Π is in zenith position in K (under the identification $x \mapsto \tau_K(x)$). Let $\alpha; \Sigma \in \mu(\mathfrak{M})^+$ be the surface sequence for α. The new link map is $\Xi; \alpha; \Sigma \mapsto \Xi; \alpha; \Pi$. To see that this defines an extension of some chain, $\alpha; \Pi$ must ac–command $\alpha; \Sigma$. For that, Π must be a proper suffix of Σ. Hence, we have to show that β can be reached from α following only maximal links. If not, however, there is a α' such that $\alpha \leq \alpha' < \beta$ and a link $\langle \alpha', \beta' \rangle$ such that $\beta' \not\leq \beta$. This is excluded by assumption. So, what we have defined is indeed a 1–step link extension of some chain. The remaining conditions are immediate to verify. Q. E. D.

Now that $\mathfrak{N}$ differs from $\mathfrak{M}$ by one link only we may say the following.

Definition 35 *Let $\mathfrak{M} = \langle M, \prec \rangle$ be an MDS, H the set of root links of $\mathfrak{M}$. A **derivation of** $\mathfrak{M}$ is a sequence $\sigma = \langle \lambda_i : i < n \rangle$ such that:*

1. *λ_i is a non root link of $\mathfrak{M}$ for every $i < n$.*
2. *$\lambda_i \neq \lambda_j$ whenever $i \neq j$.*
3. *$\langle M, H \cup \{\lambda_i : i < j+1\} \rangle$ is a link extension of $\langle M, H \cup \{\lambda_i : i < j\} \rangle$.*

*A **derivation** is a pair $\langle \mathfrak{M}, \sigma \rangle$ such that σ is a derivation of $\mathfrak{M}$.*

We shall derive the following consequence.

Corollary 36 *Let $\langle \mathfrak{M}, \sigma \rangle$ be a derivation and let $\langle M, \prec_i \rangle$, $i < n$, be the ith MDS in the derivation. Then $\prec_i^+ = \prec_0^+$ for all $i < n$.*

Proof. By induction. The claim holds for $i = 0$. Suppose it holds for i, $i < n - 1$. Then $\mathfrak{M}_{i+1}$ is obtained by adding $\lambda_i = \langle \alpha_i, \beta_i$. By definition of an MDS, $M(\alpha_i)$ is linearly ordered by $<_{i+1}$ in $\mathfrak{M}_{i+1}$. So, there is a γ such that $\alpha_i \prec_i \gamma$ and $\gamma \prec_{i+1}^+ \beta_i$. However, since $\prec_{i+1}$ is cycle free we do not have $\gamma \prec_{i+1}^* \alpha_i$, and so no path in $\mathfrak{M}_{i+1}$ from γ to β_i goes through α_i. This means that $\gamma \prec_i^+ \beta_i$, whence $\alpha_i \prec^+ \beta_i$. Q. E. D.

It follows that in a link $\langle \alpha, \beta \rangle$ we have $\alpha < \beta$. Furthermore, a link is a root link in a 1–step extension of $\mathfrak{M}$ iff it is a root link of $\mathfrak{M}$ (a fact, which can be proved directly as well).

6 Constraints on Derivations

One particular constraint on a derivation is one that concerns only the individual steps. In a sense to be made precise they may not become too long. So, first we want to establish for any given derivation what the path expressions of the individual links are. What we have to avoid is to calculate the CCS in its entirety, for its size may be exponential in the size of $\mathfrak{M}$. Fortunately, this is not necessary.

Proposition 37 *Let $\mathfrak{M}$ be an MDS and $\mathfrak{N}$ a 1–step link extension of $\mathfrak{M}$ by the link $\langle\alpha, \beta\rangle$. Let K and L be CCSs such that $M(K) \cong \mathfrak{M}$ and $M(L) \cong \mathfrak{N}$. Then the path of the link map that is added to K is Π, where $\sigma_{\mathfrak{M}}(\alpha) = \alpha; \Pi; \sigma_{\mathfrak{M}}(\beta)$. Moreover, $\Pi \neq \varepsilon$. The link map is as follows.*

$$\varphi : \Xi, \alpha; \Pi; \sigma_{\mathfrak{M}}(\beta) \mapsto \Xi; \alpha; \sigma_{\mathfrak{M}}(\beta)$$

Proof. We have seen earlier, that if $\langle\alpha, \beta\rangle$ can be added to form a link extension then the expression $\mu(\alpha; \beta)$ is actually defined and obviously unique. Moreover, notice that in K, the surface path of α is $\alpha; \Pi; \mu(\beta; \rho)$, where ρ is the root, and the surface path of β is $\mu(\beta; \rho)$. It follows that Π is the path of the link $\langle\alpha, \beta\rangle$. Moreover, if $\Pi = \varepsilon$, $\beta \succ \alpha$, which means that the link has already been there. This is a contradiction to the definition of a 1–step link extension. Q. E. D.

Hence, given a derivation, it is straightforward to compute the path expressions in polynomial time. (The time consumption is $O(n^2)$ (n the size of the MDS) for each link, hence in total $O(n^3)$ for the entire derivation.) Now there is a unique cyclic derivation in which all paths are shortest. We have claimed this in effect with Theorem 46, but now we shall establish a full proof. Furthermore, it will follow that not only are paths shortest in this derivation, this derivation will satisfy all nearness conditions if there is any derivation that does so.

Definition 38 Π *as defined in Proposition 37 is called the* ***movement path of*** λ.

Movement paths are the sort of things that are constrained by command relations. Recall that the antecedent has to c–command its trace, while the trace needs to R–command its antecedent, where R is a command relation which is determined by the type of constituent that is being moved. R command is computed using the path expression of the movement path (see

Proposition 4). In a CCS, movement paths are unique, but we shall see that a given MDS a link may have several paths of different length.

Here are now some constraints on derivations. We use the metaphor of movement. We say that $\mathfrak{C}$ is *moved* at some step in the derivation if it is the head of the chain that is extended. Effectively, this means that a constituent cannot be moved out of a derived position. This is the principle

> *Freeze.* If $\mathfrak{C} \subsetneq \mathfrak{D}$ and $\mathfrak{C}$ is in a nontrivial chain, thet $\mathfrak{D}$ is the foot of its chain.

A slightly different condition is the following.

> *Bound Traces.* Every trace must be bound.

The second says that given a set Q of nodes, any derivation must deal with constituents moving up the Q–nodes:

> *Q–Cyclicity.* Suppose that $\mathfrak{C}$ and $\mathfrak{D}$ are in nontrivial chains. If $\mathfrak{C}$ does not Q–command $\mathfrak{D}$, then $\mathfrak{C}$ is moved before $\mathfrak{D}$.

The next principle is formulated using MDSs and CCSs. Since the length of links is the same in all derivation of the same CCSs, there is nothing to choose from. However, suppose that we are not interested in the CCSs but in the MDSs.

> *Shortest Steps.* A derivation of an MDS $\mathfrak{M}$ must be such that the movement path for any given link is a subset of the movement path of that link in any other derivation.

Lemma 39 *Let $\mathfrak{M}$ be an MDS and $\mathfrak{N}$ a 1–step link extension of $\mathfrak{M}$ by the link $\langle\alpha,\beta\rangle$. Let K and L be CCSs such that $M(K) \cong \mathfrak{M}$ and $M(L) \cong \mathfrak{N}$. Let Π be the movement path of the link $\langle\alpha,\beta\rangle$, and φ the link map. Then the following holds for the surface paths.*

1. *If $\sigma_{\mathfrak{M}}(\gamma) = \Xi;\alpha;\Pi;\Sigma$ then $\sigma_{\mathfrak{N}}(\gamma) = \varphi(\sigma_{\mathfrak{M}}(\gamma)) = \Xi;\alpha;\Sigma$.*
2. *Otherwise $\sigma_{\mathfrak{N}}(\gamma) = \sigma_{\mathfrak{M}}(\gamma)$.*

So, the surface path of an element in the link extension is a circumfix of the surface path of $\mathfrak{M}$. Here, $\vec{x} \in A^*$ is a circumfix of $\vec{y}$ if there are $\vec{u},\vec{v},\vec{w} \in A^*$ such that $\vec{x} = \vec{u}\,\vec{w}$ and $\vec{y} = \vec{u}\,\vec{v}\,\vec{w}$. Let us call $\vec{v}$ the **nucleus** of $\vec{y}$ with respect to the circumfix $\vec{x}$. (The nucleus is in general not unique, but in the present case it is.) The nucleus is exactly the movement path of the link.

7 Classifying Link Interactions

The idea that we shall follow is that we take an MDS and try to remove the non deep links one by one such that the inverse process, the addition of the links, is a derivation. We shall look for a derivation for a given MDS that satisfies certain locality conditions. For that, we try to find derivations in which the paths are as short as possible. We know that it is quite straightforward to find a derivation in which paths are as long as possible, namely the so called *Freeze*–derivation. After having found the Freeze–derivation we try to commute the links and reduce the lengths of paths. First, we shall focus on pairs of links. We shall how they can lie with respect to each other and then calculate how the derivation can look like in these cases.

We shall proceed to the classification of link geometries. Let $\lambda = \langle \alpha, \beta \rangle$, $\lambda' = \langle \alpha', \beta' \rangle$ be distinct (non root) links. Then $\alpha < \beta$ and $\alpha' < \beta'$. Write $\alpha \shortparallel \alpha'$ if α and α' are incomparable via $\leq$.

1. $\alpha \shortparallel \alpha'$. Then we say that λ and λ' are **parallel** and also write $\lambda \shortparallel \lambda'$. If the links are not parallel, we may assume without loss of generality that $\alpha \leq \alpha'$.

2. $\alpha = \alpha'$. Then without loss of generality $\beta < \beta'$. (Since the links are distinct, $\beta \neq \beta'$.) In this case we say that is **coeffluent with** λ'.

3. $\alpha < \alpha'$. Then either $\beta \leq \beta'$ or $\beta' \leq \beta$. (Notice that $\leq = \prec_0^*$, and so in an MDS if $\beta, \beta' \geq \alpha$ they are comparable via $\prec_0^*$.)

   (a) $\beta \leq \alpha'$. We say that λ is **lower than** λ'.

   (b) $\alpha' < \beta < \beta'$. We say that the two links **cross**.

   (c) $\beta = \beta'$. This does not arise in a binary branching structure.

   (d) $\beta' < \beta$. Then we call the links **concentric**. Also we say that λ' is **inside of** λ.

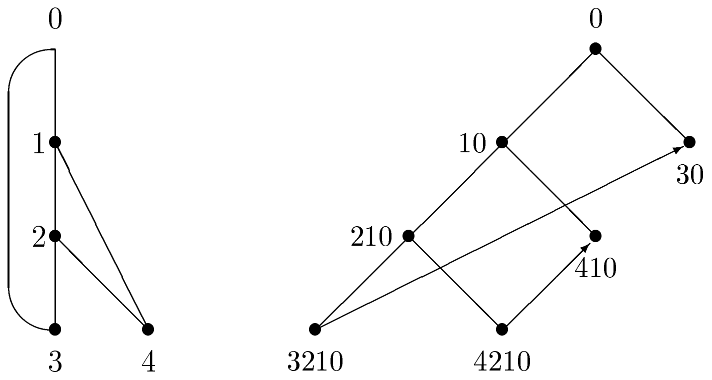
It is easily checked that the list is exhaustive. Now we shall check whether or not we can insert one link before the other and if so, what effect this has.

7.1 Parallel Links

Let $\mathfrak{M}_1 := \langle \{0, 1, 2, 3, 4\}, \prec_0 \rangle$ with

$$\prec_1 := \{\langle 3, 2 \rangle, \langle 2, 1 \rangle, \langle 1, 0 \rangle, \langle 3, 0 \rangle, \langle 4, 2 \rangle, \langle 4, 1 \rangle\}$$

Figure 1: Parallel Links



(See Figure 1 on the left.) Then the root links are $\langle 3, 2\rangle$, $\langle 3, 2\rangle$, $\langle 2, 1\rangle$, $\langle 1, 0\rangle$ and $\langle 4, 2\rangle$. The non root links are $\langle 4, 1\rangle$ and $\langle 3, 0\rangle$. These links are parallel since $3 \parallel 4$. The paths are

$$4210, 3210, 410, 210, 10, 30, 0$$

This defines the tree for $\langle \mathfrak{T}_1, K_1\rangle$. The nontrivial equivalence classes are $[4210]_{K_1} = \{4210, 410\}$ and $[3210]_{K_1} = \{3210, 30\}$. There are two ways to derive $\mathfrak{M}_0$. (A) We add the link $\langle 4, 1\rangle$ before the link $\langle 3, 0\rangle$. (B) We add the link $\langle 3, 0\rangle$ before the link $\langle 4, 1\rangle$. The movement paths and the link maps coincide in the Case (A) and (B). For the link $\langle 4, 1\rangle$ the link map is $4210 \mapsto 410$. If there is a constituent attached to 3, we would have the map $\Xi; 4210 \mapsto \Xi; 410$. Hence the movement path is 2. Now for the link $\langle 3, 0\rangle$ the link map is $\Xi; 3210 \mapsto \Xi; 30$ so the movement path is 21.

It turns out that if two links are parallel, the movement paths are identical for all elements, no matter which order we choose. (See Figure 1 to the right.)

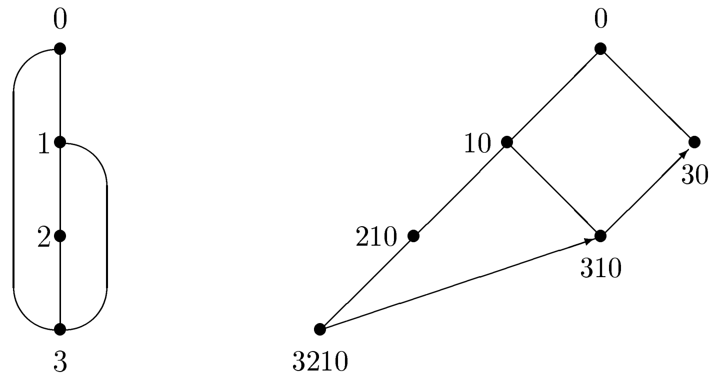
7.2 Coeffluent Links

See Figure 2 Let $\mathfrak{M}_2 := \langle \{0, 1, 2, 3\}, \prec_2\rangle$ with

$$\prec_2 := \{\langle 3, 2\rangle, \langle 2, 1\rangle, \langle 1, 0\rangle, \langle 3, 1\rangle, \langle 3, 0\rangle\}$$

Then the root links are $\langle 3, 2\rangle$, $\langle 3, 2\rangle$, $\langle 2, 1\rangle$, and $\langle 1, 0\rangle$. The non root links are $\langle 3, 1\rangle$ and $\langle 3, 0\rangle$. These links are not parallel. We have a case of coeffluent

Figure 2: Coeffluent Links



links. The paths are

$$3210, 310, 30, 210, 10, 0$$

This defines the tree for $\langle \mathfrak{T}_2, K_2 \rangle$. The only nontrivial equivalence class is $[3210]_{K_2} = \{3210, 310, 30\}$. There is only one way to derive $\mathfrak{M}_2$, namely we add the link $\langle 3, 1 \rangle$ before the link $\langle 3, 0 \rangle$.

The movement paths and the link maps are as follows. For the link $\langle 3, 1 \rangle$ the link map is $\Xi; 3210 \mapsto \Xi; 310$. Hence the movement path is 2. Now for the link $\langle 3, 0 \rangle$ the link map is $\Xi; 310 \mapsto \Xi; 30$ so the movement path is 1.

Suppose we first add $\langle 3, 0 \rangle$. Then the link map is $\Xi; 3210 \mapsto \Xi; 30$, with movement path 21. The second link map would be $\Xi; 30 \mapsto \Xi; 310$, which is excluded, since 310 does not c–command (and therefore not ac–command) 30.

So what have here (translated into derivations) is the case of an extension of a chain that has more than one member.

7.3 Hierarchically Ordered Links

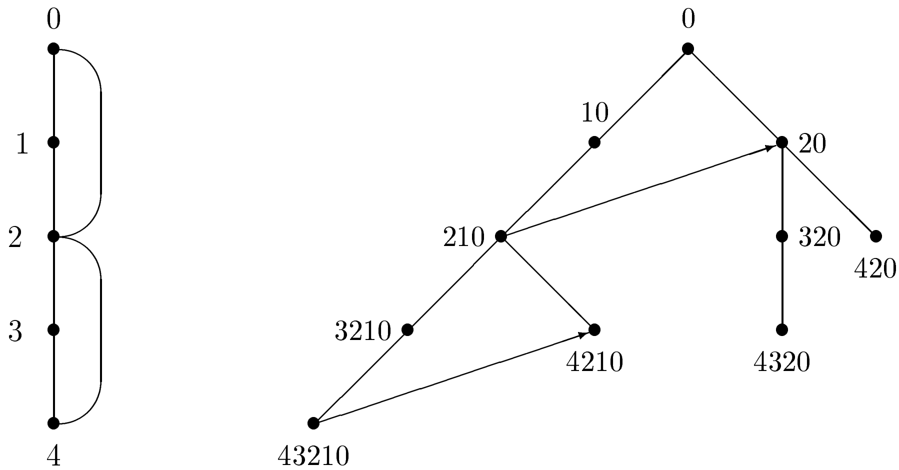
See Figure 3. Let $\mathfrak{M}_3 := \langle \{0, 1, 2, 3, 4\}, \prec_3 \rangle$ with

$$\prec_3 := \{\langle 4, 3 \rangle, \langle 3, 2 \rangle, \langle 2, 1 \rangle, \langle 1, 0 \rangle, \langle 4, 2 \rangle, \langle 2, 0 \rangle\}$$

Then the root links are $\langle 4, 3 \rangle$, $\langle 3, 2 \rangle$, $\langle 3, 2 \rangle$, $\langle 2, 1 \rangle$, and $\langle 1, 0 \rangle$. The non root links are $\langle 4, 2 \rangle$ and $\langle 2, 0 \rangle$. These links are not parallel since $4 \leq 2$. The paths are

$$43210, 4210, 420, 40, 3210, 320, 210, 20, 0$$

Figure 3: Hierarchically Ordered Links



This defines the tree for $\langle \mathfrak{T}_3, K_3 \rangle$. The nontrivial equivalence classes are $[43210]_{K_3} = \{43210, 4210, 420, 40\}$, $[3210]_{K_3} = \{3210, 320\}$ and $[210]_{K_3} = \{210, 20\}$. There is only one way to derive $\mathfrak{M}_2$, namely we add the link $\langle 4, 2 \rangle$ before the link $\langle 2, 0 \rangle$.

The movement paths and the link maps are as follows. For the link $\langle 4, 2 \rangle$ the link map is $43210 \mapsto 4210$. If there is a constituent attached to 4, we would have the map $\Xi; 43210 \mapsto \Xi; 4210$. Hence the movement path is 3. Now for the link $\langle 2, 0 \rangle$ the link map is $\Xi; 210 \mapsto \Xi; 20$ so the movement path is 1.

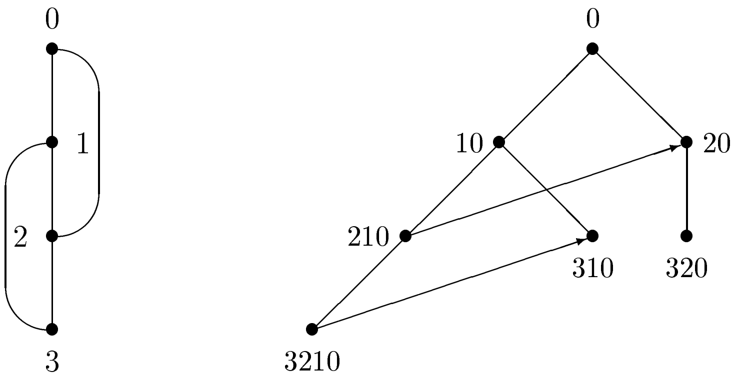
In fact, if we are just interested in movement paths, we may commute the links. If we first add the path $\langle 2, 0 \rangle$, the surface path of 4 becomes 4320. If we then add the link $\langle 4, 0 \rangle$ it becomes 420. Although on the paths there is no problem, the ordering is introduced for abstract reasons (it does not mirror into CCS derivations).

What we have here is a case where there is apparently a choice between two kinds of derivations. However, it is agreed that the link must be added prior to the higher link. (This is enshrined in the postulate *Liberation*.)

7.4 Crossed Links

See Figure 4. Let $\mathfrak{M}_4 := \langle \{0, 1, 2, 3\}, \prec_4 \rangle$ with

Figure 4: Crossed Links



$$\prec_4 := \{\langle 3,2\rangle, \langle 2,1\rangle, \langle 1,0\rangle, \langle 3,1\rangle, \langle 2,0\rangle\}$$

Then the root links are $\langle 3,2\rangle$, $\langle 3,2\rangle$, $\langle 2,1\rangle$, and $\langle 1,0\rangle$. The non root links are $\langle 3,1\rangle$ and $\langle 2,0\rangle$. These links do not commute since $3 \leq 2$. We have a case of crossed links. The paths are

$$3210, 320, 310, 210, 20, 10, 0$$

This defines the tree for $\langle \mathfrak{T}_4, K_4\rangle$. The nontrivial equivalence classes are $[3210]_{K_4} = \{3210, 310, 320\}$ and $[210]_{K_4} = \{210, 20\}$. There is only one way to derive $\mathfrak{M}_4$, namely we add the link $\langle 3,1\rangle$ before the link $\langle 2,0\rangle$.

The movement paths and the link maps are as follows. For the link $\langle 3,1\rangle$ the link map is $\Xi; 3210 \mapsto \Xi; 310$. Hence the movement path is 2. Now for the link $\langle 2,0\rangle$ the link map is $\Xi; 210 \mapsto \Xi; 20$ so the movement path is 1.

Suppose we first add $\langle 2,0\rangle$. Then the surface path of 3 is 320. The link map would be $\Xi; 320 \mapsto \Xi; 310$. There is no movement path. In particular, 310 does not ac–command 320, so we do not have a proper link map.

Translated into CCSs, this means that we have a case of remnant movement.

7.5 Concentric Links

See Figures 5 and 6. Let $\mathfrak{M}_5 := \langle \{0,1,2,3,4\}, \prec_5\rangle$ with

$$\prec_5 := \{\langle 4,3\rangle, \langle 3,2\rangle, \langle 2,1\rangle, \langle 1,0\rangle, \langle 4,0\rangle, \langle 3,1\rangle\}$$

Figure 5: Concentric Links: Freeze Movement

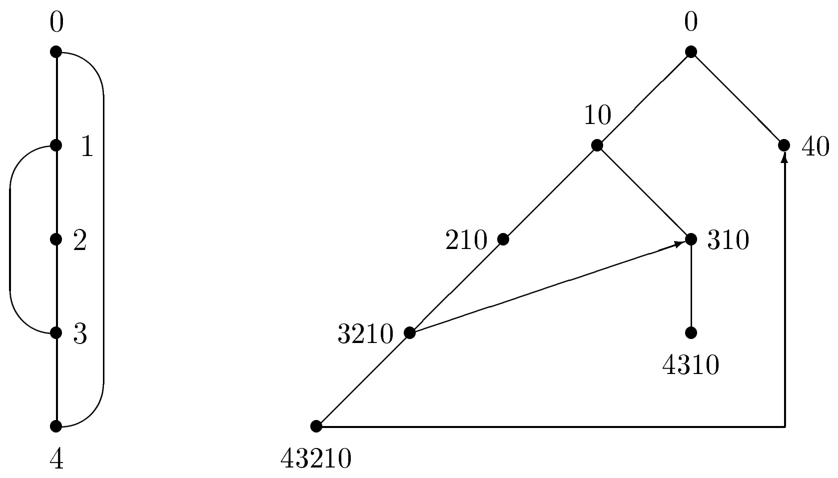
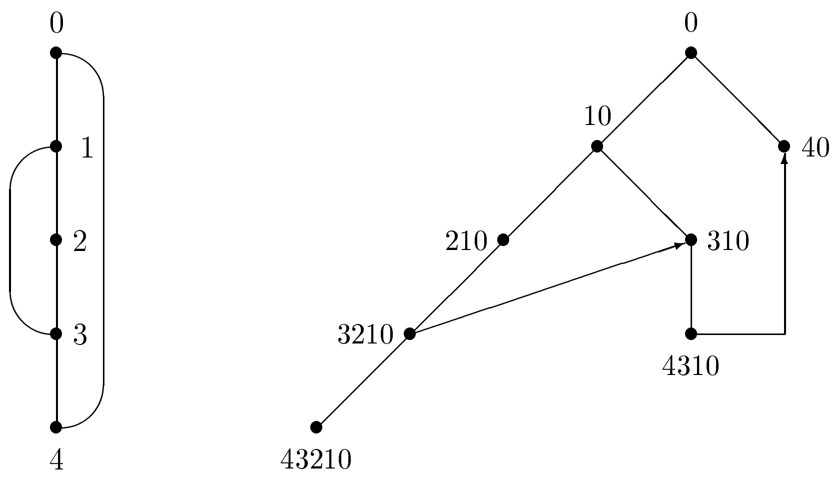


Figure 6: Concentric Links: Piggy Backing



Then the root links are $\langle 4, 3\rangle$, $\langle 3, 2\rangle$, $\langle 3, 2\rangle$, $\langle 2, 1\rangle$, and $\langle 1, 0\rangle$. The non root links are $\langle 4, 0\rangle$ and $\langle 3, 1\rangle$. The paths are

$$43210, 4310, 40, 3210, 310, 210, 10, 0$$

This defines the tree for $\langle \mathfrak{T}_5, K_5\rangle$. The nontrivial equivalence classes are $[43210]_{K_5} = \{43210, 4310, 40\}$ and $[3210]_{K_5} = \{3210, 310\}$. There are two ways to derive $\mathfrak{M}_3$, namely (A) we add the link $\langle 4, 0\rangle$ before the link $\langle 3, 1\rangle$ and (B) we add the link $\langle 3, 1\rangle$ before the link $\langle 4, 0\rangle$.

The movement paths and the link maps are as follows. Case (A). For the link $\langle 4, 0\rangle$ the link map is $43210 \mapsto 40$. The movement path is 321. If there is a constituent attached to 4, we would have the map $\Xi; 43210 \mapsto \Xi; 40$. Now for the link $\langle 3, 1\rangle$ the link map is $\Xi; 3210 \mapsto \Xi; 310$ so the movement path is 2.

Case (B). For the link $\langle 3, 1\rangle$ the link map is $\varphi : \Xi; 3210 \mapsto \Xi; 310$ with movement path 2. Notice that the surface paths have changed. Now the surface path of 4 is $\varphi(43210) = 4310$. The surface path of 0 is 0. Hence the link map is now $\Xi; 4310 \mapsto \Xi; 40$, with movement path 31.

If we compare (A) and (B) we see that in the (B) the path of the link $\langle 4, 0\rangle$ is 31 rather than 321. It has been shortened by the movement path of $\langle 3, 1\rangle$. This is the *only* genuine case of choice that would enable us to shorten paths. The first derivation takes long paths, and is *Freeze* compatible. The second is *Shortest Steps* compatible. Here, the lower element takes a free ride upstairs (piggy backing).

8 Enumerating Derivations

Now that we have looked at the link geometry, we shall turn to derivations. It is clear first of all that the question whether or not a link can be added at a given moment depends on its configuration with respect to all other links. Hence, we must now take care of that as well. This is however not hard to do.

Definition 40 *Let $\lambda = \langle \alpha, \beta\rangle$ and $\lambda' = \langle \alpha', \beta'\rangle$ be links. Then we say that λ* ***precedes*** *λ', in symbols $\lambda \ll \lambda'$, if either*

1. *λ is lower than λ', or*
2. *λ' and λ are coeffluent and $\beta < \beta'$, or*

3. λ' and λ cross and $\alpha < \alpha'$.

The following is clear from the previous section.

Lemma 41 *Let $\langle \mathfrak{M}, \sigma\rangle$ be a derivation of $\mathfrak{M}$, $\sigma = \langle \lambda_i : i < n\rangle$. Then if $\lambda_i \ll \lambda_j$, $i < j$.*

Theorem 42 *Let $\mathfrak{M}$ be an MDS, $\sigma = \langle \lambda_i : i < n\rangle$ an enumeration of the non root links of $\mathfrak{M}$. Then $\langle \mathfrak{M}, \sigma\rangle$ is a derivation if and only if $\lambda_i \ll \lambda_j$ implies $i < j$.*

Proof. We use Theorem 34. Suppose, we want to add the link $\lambda_i = \langle \alpha_i, \beta_i\rangle$. Then we must make sure that β_i is not derived in $\langle M, H \cup \{\lambda_j : j < i\}\rangle$ and that the link does not cross or is lower than any of the previous links. If it does, then $\lambda_i \ll \lambda_j$, contradiction. So, we must show that $M(\gamma)$ has exactly one element for all $\gamma \geq \beta$. Suppose not. Then there is a $\gamma \geq \beta$ such that $\gamma = \alpha_j$ for some $j < i$. Look at the link $\lambda_j = \langle \alpha_j, \beta_j\rangle$. We have $\alpha_j \geq \beta_i > \alpha_i$. Then the link λ_j is higher than λ_i. This is excluded since $\lambda_i \ll \lambda_j$. Q. E. D.

So, we have by means of $\ll$ a complete map of the possible derivations. Any linear order extending $\ll$ gives rise to a derivation, and moreover any derivation induces a linear order on the links that is compatible with $\ll$. So, how do we decide which is the shortest? To this end, we first look at parallel links. Obviously, if we exchange adjacent parallel links in a derivation, the paths remains the same. However, it would not do to simply exchange a pair of parallel links, since this may not even result in a derivation. The general definition is therefore this one.

Definition 43 *Let $\langle \mathfrak{M}, \sigma\rangle$ and $\langle \mathfrak{M}, \sigma'\rangle$ be derivations, and assume that $\sigma = \langle \lambda_i : i < n\rangle$ and $\sigma' = \langle \lambda_{\pi(i)} : i < n\rangle$ for some permutation π of the numbers $< n$. Then say that the derivations are* ***equivalent*** *if the following holds: If λ_i and λ_j are concentric and λ_i is inside λ_j, then from $i < j$ follows $\pi(i) < \pi(j)$ and conversely.*

Lemma 44 *Suppose that σ and σ' are equivalent derivations of the same structure. Then the movement paths of the links are the same in both derivations.*

So, there is nothing to choose between equivalent derivations. This leaves only one configuration where we can manipulate the lengths: concentric links.

Here we have $\alpha < \alpha' < \beta' < \beta$. We may either decide to add first the link $\langle\alpha, \beta\rangle$ and then the link $\langle\alpha', \beta'\rangle$. The second turns out to be the best move in that it shortens the paths for all elements. Call λ' a **transporter** for λ if (a) $\alpha < \alpha' < \beta' < \beta$, (b) there is no λ'' such that $\lambda \ll^+ \lambda'' \ll^+ \lambda'$. It is clear from the preceding calculations that if λ' is a transporter for λ, then links are shorter (though not always strictly) for all links if λ' precedes λ in the derivation.

Lemma 45 *Let $\mathfrak{M}$ be an MDS and λ a transporter of λ'. Let σ and σ' be two derivations such that (a) λ' precedes λ in σ, (b) σ' results from σ by exchanging λ and λ'. Further, let $S_\sigma(\mu)$ be the path set of μ in σ, $S_{\sigma'}(\mu)$ the path set of μ in σ'. Then $S_{\sigma'}(\mu) \subseteq S_\sigma(\mu)$ for all non root links μ.*

Hence we have the following theorem.

Theorem 46 *The following holds for every MDS.*

1. *Up to equivalence, there is a unique derivation for a given MDS such that λ precedes λ' iff λ' is a transporter for λ. This derivation satisfies Freeze.*

2. *Up to equivalence, there is a unique derivation for a given MDS such that λ precedes λ' iff λ is a transporter for λ'. This derivation satisfies Shortest Links.*

The consequences are immediate. If nearness conditions on movement are formulated using definable command relations (such is the case with subjacency) then there is a derivation satisfying it if and only if the *Shortest Links* derivation satisfies it. This follows from the subset principle.

Let us close with the principle *Bound Traces*.

Theorem 47 *Let $\mathfrak{M}$ be an MDS. $\mathfrak{M}$ has a derivation satisfying Bound Traces iff there are no crossed links. In this case* ***every*** *derivation of $\mathfrak{M}$ satisfies Bound Trace.*

So, unbound traces occur whenever there is no remnant movement.

Our task is not yet finished. We need to establish the movement paths of the individual links. To that end, notice the following. Let $\lambda = \langle\alpha, \beta\rangle$ be a link. Then compute the path $\mu^-(\alpha; \beta)$ in $\mathfrak{M}$ with λ eliminated. (It can be defined as follows: let β^- be element directly lower than β in $M(\alpha)$. Then $\mu^-(\alpha; \beta) := \alpha; \mu(\beta^-; \beta)$.) It turns out that this is exactly the movement path in a *Shortest Steps* derivation. For let $\lambda' = \langle \alpha, \beta' \rangle$ be any link such that $\beta' < \beta$. Then by our obeservations, λ' precedes λ. Also, if $\alpha < \gamma < \beta$ and $\langle \gamma, \beta' \rangle$ is a link such that $\beta' \leq \beta$, then this link will precede λ. So, the surface path between α and β in the structure derived just before λ is added is exactly as described. So, locality conditions can in fact be computed very fast for a *Shortest Steps* derivation.

Theorem 48 *Let $\mathfrak{M}$ be an MDS and $\lambda = \langle \alpha, \beta \rangle$ be a non root link. Then the movement path of λ in a derivation satisfying* Shortest Steps *is exactly $\mu^-(\alpha; \beta)$.*

9 Cyclicity

Cyclicity is the condition that lower links precede higher links. That condition has to be precisified. One natural precisification is the following, which we spell out first for CCSs.

Definition 49 *Let $Q \subseteq A$. We say that $\langle \mathfrak{C}, \varphi, \mathfrak{D} \rangle$ is Q–**inferior to** $\langle \mathfrak{C}', \varphi', \mathfrak{D}' \rangle$ with $\mathfrak{C} = \downarrow x$, $\mathfrak{C}' = \downarrow x'$ if there is a node y such that $x < y < x'$ and $\ell(y) \in C$.*

This definition can also be rendered using command relations. We may think of a partitioning of the tree in shells in the following way. Say that x and y are Q–**equidistant** if they Q–command each other. If x Q–commands y but not conversely, say that y **antisymmetrically** Q**–commands** x. Then the above condition says that if $\downarrow y$ and $\downarrow x$ are moved and x antisymmetrically Q–commands y then $\downarrow y$ must be moved before $\downarrow x$. Cyclicity is therefore a constraint on a derivation, however, one which can be checked by comparing pairs of links.

Cyclicity is easily implemented.

Definition 50 *Let $\mathfrak{M}$ be an MDS, and $\lambda = \langle \alpha, \beta \rangle$ and $\lambda' = \langle \alpha', \beta' \rangle$ non root links. If λ and λ' are concentric, write $\lambda \rtimes_Q \lambda'$ if $\mu(\alpha; \alpha')$ contains an occurrence of a Q–node.*

The following is easily shown on the basis of Theorem 42.

Theorem 51 *Let $\mathfrak{M}$ be an MDS, $\sigma = \langle \lambda_i : i < n \rangle$ an enumeration of the non root links of $\mathfrak{M}$. Then $\langle \mathfrak{M}, \sigma \rangle$ is a cyclic derivation if and only if*

1. *$\lambda_i \ll \lambda_j$ implies $i < j$.*

2. *If* $\lambda_i \rtimes_Q \lambda_j$ *then* $i < j$.

Corollary 52 *Let* $\mathfrak{M}$ *an MDS. Then up to equivalence there is a unique derivation satisfying Q–cyclicity.*

So, *Cyclicity* and *Shortest Steps* are opposing principles. In fact, *Freeze* is identical to *A–Cyclicity*. For if every node is cyclic, the order of nonparallel links is totally fixed, and in the Freeze–order.

10 Conclusion

We have studied the space of possibly derivations for a given MDS (or TCS, for that matter). Given an MDS $\mathfrak{M}$, there exist in the worst case exponentially many derivations for $\mathfrak{M}$. (If all links are concentric.) It is however possible to map the space of all possible derivations in polynomial time. This is done in the form of an ordering relation on the links which must be respected by all derivations. Moreover, using this representation it is possible to compute the length of link maps. This allows to establish very swiftly whether a (cyclic) derivation exists that satisfies certain nearness conditions.

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